POLITICAL ECONOMY

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A Political Theory of Progressive Income Taxation

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I. Introduction

Progressive income taxation is found in all developed and in many developing countries. In most of these countries both average and marginal income tax rates increase with the level of income. Yet this ubiquitous phenomenon has proved troublesome for economists and social scientists. Despite numerous attempts to make the case for or against progressivity, and many strong statements on both sides, the rational case for progressivity has proved elusive.

Over a generation ago, Blum and Kalven (1953) pointed out that most arguments for progressivity have a weak foundation. Many rely on comparisons of marginal utility of income across individuals, an assumption that economists reject along with other interpersonal comparisons of utility. After considering the arguments, Blum and Kalven (p. 71) concluded that the strongest case for progressive taxes depends on the argument that progressivity provides revenues for redistribution through the government budget. There is now broad agreement that the case for redistribution and tax progressivity cannot be made on strictly economic, nonpolitical grounds.[1]

Despite this broad agreement, much recent research on tax progressivity focuses on the conditions that would lead to the choice of progressive taxes as an optimal form of taxation. This work is normative, not positive, and much of it is based on the utilitarian principle of maximizing the sum of individual utilities.

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1. See the comments by James Tobin, Allen Wallis, Oswald Brownlee, Norman Ture, and Richard Musgrave in Campbell (1977).

Pigou (1947) showed that this principle leads to extreme progressivity; his optimal tax policy is full equalization of after-tax incomes. Mirrlees (1971) showed that Pigou's conclusion changes considerably when there are incentive effects of taxation on the choice between labor and leisure. Tax rates are much lower, and tax schedules are either linear or rates fall as income rises (Mirrlees, 1971; Phelps, 1973; Sadka, 1976).[2] Atkinson (1973) imposed a social utility function: the government chooses to alter the distribution of income by lowering inequality, as Simons (1938, pp. 18–19) had urged. Progressivity can be obtained in this case if the government is willing to move the economy to a Pareto inferior position.

The optimal tax literature typically imposes a utility function and derives the tax function. An alternative approach taken by Romer (1975) and Roberts (1977) is to specify a tax function and allow the voters to choose the parameters of this function through majority rule. Romer (1975) showed that, if individuals differ in ability, the decisive voter is the individual with median ability. The decisive voter chooses increasing average progressivity (Romer, 1975), if the utility function is Cobb–Douglas. Meltzer and Richard (1981) extended this result for net redistribution to a large class of utility functions and provided evidence (Meltzer & Richard, 1983) that the model is broadly consistent with U.S. data.

Tax schedules and so-called effective tax rates typically rise with the level of income. Marginal tax rates often rise also. Table 5-1 compares the effective tax rate paid by a family with two children that earns the mean level of income to the tax rates paid by a comparable family that earns two or four times mean income. In most countries, the average effective tax rate increases as income rises.[3]

Table 5-1 also shows that countries have very different tax functions. Denmark, Germany, and Sweden have similar per capita income and all three have relatively high average tax rates at mean income. Progressivity differs, however; it is greater in Sweden and Denmark than in Germany. Australia, Japan, and Canada have relatively low tax rates and, again, marked differences in progressivity. The Australian increase in tax rates with income is similar to Sweden's, but the average rates are much lower.

Differences in tax rates are often associated with differences in spending for redistribution. Meltzer and Richard (1981), using a linear tax function, showed that the decisive voter's choice of per capita transfer payments determines the tax rate. Net tax payments are negative at low incomes and for nonworkers, and rates rise with the level of income in their analysis.

2. An early survey of this literature is in Atkinson (1973). Mirrlees' (1971, p. 186) own summary is: "The optimum tax schedule depends upon the distribution of skills in such a complicated way that it is not possible to say in general whether marginal rates should be higher for high income, low income or intermediate income groups."

Linearity of the optimal schedule extends to the case in which individuals respond to higher tax rates by working in an untaxed sector of the economy (Kramer & Snyder, 1983, 1984).

3. We believe this is true in the United States also, although the table is restricted to countries for which the Organization for Economic Cooperation and Development (OECD) attempts to provide comparable data.

Table 5-1. Average tax rates of a family with two children filing a joint tax return at various income levels, 1974[a]

Country	Normalized Gross Income Level 100%	200%	400%
Australia	7.4	20.1	35.7
Austria	13.2	18.6	24.1
Belgium	15.2	22.5	30.4
Canada	9.8	19.5	28.4
Denmark	31.0	43.3	52.7
Finland	22.9	32.5	43.5
France	8.4	12.8	16.9
Germany	22.9	29.5	35.4
Ireland	15.7	24.1	36.5
Italy	7.7	12.4	23.7
Japan	8.4	12.1	18.2
New Zealand	15.6	24.7	36.0
Norway	21.2	31.3	44.0
Spain	7.0	8.4	15.0
Sweden	24.4	36.2	50.5
Switzerland	17.3	25.9	34.6
United Kingdom	13.7	26.0	30.9

[a]Gross income is expressed as a percentage of an average production worker's earnings. The tax rates are for a family income that is contributed in equal shares by both spouses and they include both personal income taxes and social security contributions. Data are available in the source for other earning profiles.

Source: Committee on Fiscal Affairs, Organization for Economic Cooperation and Development, (1978). *The Tax/Benefit Position of Selected Income Groups in OECD Member Countries 1972—1976.* Paris: OECD, Table 16(b), p. 110.

This chapter provides a positive theory of progressivity for marginal and average tax rates within a majority-rule framework by using a tax function that permits marginal progressivity, linearity, or regressivity. The analysis suggests some reasons for observed differences in marginal and average tax rates.

The analysis develops a set of conditions under which majority voting implies marginal progressivity. We assume that each person has the same utility function, but that people differ in ability and therefore in productivity. Under majority rule, the choice of progressivity depends, partially, on the response of labor supply to tax rates. If people with high ability show small response of labor supply to tax rates, majority rule is likely to produce marginal progressivity. Marginal progressivity is not restricted to this case, however. Even when the response of work effort to an increase in tax burden is not systematically related to income levels, the decisive voter can choose marginal progressivity. He is more likely to do so the higher the variance of gross incomes and the more skewed to the right the distribution of gross incomes. Studies of the distribution of income show that the distribution is indeed skewed to the right. The variance and the degree of positive skewness in the distribution of gross incomes are in turn larger the larger the variance and the degree of positive skewness in the distribution of abilities.

To allow the majority-rule process to pick a (possibly) progressive tax schedule it is necessary to extend the family of linear tax schedules used in previous literature to a three-parameter family. The government's budget constraint requires that the budget be balanced. This constraint determines one of these parameters as a function of the other two, and majority rule picks the remaining two parameters out of the set of feasible pairs of such parameters. It is well known that majority voting over a multidimensional issue space of this kind may induce collective intransitivities that preclude the existence of a stable, unique majority winner. This is an aspect of Arrow's (1951) impossibility theorem. Hence it is necessary to determine whether a majority-winning tax schedule exists. This task logically precedes that of finding conditions for progressivity since, in the absence of a majority winner, majority rule does not produce a well-defined choice of tax schedule.

We show that if the ranking of incomes is independent of tax schedules (which is the case when both consumption and leisure are normal goods), the set of local majority-winning schedules is not empty. Moreover this set contains the set of feasible schedules most preferred by the individual who is at the median of the distribution of abilities. Hence the individual with median ability is locally decisive. This individual is also globally decisive for a wide range of utility functions, provided the proportion of individuals with intermediate levels of ability is sufficiently large.

Section II analyzes the labor-leisure choice of individuals with different abilities who face a tax schedule with a given degree of progressivity or regressivity. Section III introduces the government budget constraint and shows that, when the ranking of gross incomes is independent of the tax schedule, the individual with median ability is locally decisive for the choice of tax schedule. Conditions under which this individual is also globally decisive over all feasible tax schedules are discussed in Section IV. Section V characterizes the choice of tax schedule by the person with median ability who works and also provides conditions under which he will pick a progressive tax schedule. Section VI briefly considers the same issues when the decisive voter does not work. Section VII shows, by means of an example, that conditions that are likely to produce progressivity when the budget is used for redistribution of income are also likely to lead to progressivity when the budget is used to finance a public good. A conclusion summarizes main results.

II. The Private Economy

The economy consists of a large number of individuals who differ in ability and therefore in their real wage rate. Each individual takes his wage rate and the tax schedule as given and chooses the amount of leisure, work, and consumption to maximize utility. Utility of the representative individual is given by a strictly concave function $u(c, l)$ of consumption c and leisure l. Consumption is a normal good, and the marginal utility of consumption or leisure is infinite when the level of either consumption or leisure is zero.

Individual incomes reflect differences in individual productivity and the use of a common, constant returns-to-scale technology to produce the consumption good. An individual with productivity x earns pretax income y;

$$y(x) = xn(x) \tag{1}$$

where $n(x)$ is the amount of work he supplies. Each individual is endowed with one unit of time that he can allocate either to leisure, $l(x)$, or to the production of the consumption good, so $l(x) = 1 - n(x)$.

Tax revenues finance a fixed level of government expenditures G. The total tax paid by an individual with gross income y is

$$T(y) = -r + \tau y + \alpha y^2 \tag{2}$$

where r, τ, and α are parameters of the tax schedule that are determined by the political process. The corresponding marginal tax schedule is

$$T'(y) = \tau + 2\alpha y \tag{3}$$

The parameter α measures the degree of marginal progressivity of the tax schedule. The marginal tax rate increases with income when α is positive and decreases with income when α is negative.[4] For $\alpha = 0$, equation (2) reduces to the widely used linear income tax schedule (Roberts, 1977; Romer, 1975; Sheshinski, 1972), and the marginal rax rate is constant at τ. A positive r corresponds to the case in which low-income people get a subsidy by means of a negative income tax or cash transfer. This is the case discussed by Meltzer and Richard (1981). When $\alpha = 0$ and $r > 0$, our model reduces to theirs.

We restrict the tax schedule in three ways. First, negative marginal tax rates are excluded. Second, marginal tax rates are smaller than 100 percent. Third, no individual pays more than 100 percent in taxes, so $r \geqslant 0$. Equation (4) summarizes the first two restrictions. The restrictions

$$0 \leqslant T'(y) = \tau + 2\alpha y < 1 \tag{4}$$

implicitly impose upper and lower bounds on the degree of marginal progressivity ($\alpha > 0$) and marginal regressivity ($\alpha < 0$), respectively.[5]

4. A sufficient condition for both marginal and average progressivity is $\alpha > 0$ and $r \geqslant 0$.

5. Let $x_u = \max x$ be the highest ability level in the population. Since each individual supplies at most one unit of labor, the maximum income of any individual is x_u. A sufficient condition for the inequalities in equation (4) is:

$$0 \leqslant \tau + 2\alpha x_u \leqslant 1 \quad \text{for all } y \leqslant x_u$$

This expression imposes the following bounds on α

$$\frac{-\tau}{2x_u} \leqslant \alpha \leqslant \frac{1-\tau}{2x_u}$$

There is no saving; consumption equals disposable income as shown in (5).

$$c(x) = r + (1 - \tau)xn - \alpha(xn)^2 \tag{5}$$

Given the wage rate, x, and the tax parameters r, τ, and α, individuals choose the allocation of their time and their consumption by solving

$$\max_n u[r + (1 - \tau)xn - \alpha(xn)^2, 1 - n] \tag{6}$$

When $r = 0$, there is no redistribution. Everyone works because we have assumed that the marginal utility of consumption is infinite when consumption is zero. Since the the marginal utility of leisure is also infinite at zero leisure, the solution to the problem in (6) is an internal one for all x. The first-order condition is

$$x[1 - T'(y)]u_c[c(x), 1 - n(x)] - u_l[c(x), 1 - n(x)] = 0 \tag{7}$$

where

$$1 - T'(y) \equiv 1 - \tau - 2\alpha xn \tag{8}$$

and $c(x)$ is given by (5). Equation (7) determines n as a function of individual productivity x and the tax parameters r, τ, and α.[6]

Individuals with productivity below a minimum level, denoted x_0, do not work. Their earned income is zero, and their consumption is r. The value of x_0, which divides the population into workers and nonworkers, is found from (7) to be

$$x_0 = \frac{u_l(r, 1)}{(1 - \tau)u_c(r, 1)} \tag{9}$$

The value of x_0 depends on r and τ but not on α since, at x_0, the person chooses full-time leisure and $xn = 0$. The effects of r and τ on x_0 are given by (10) and (11).

$$\frac{\partial x_0}{\partial r} = \frac{u_{lc}(r, 1) - (1 - \tau)x_0 u_{cc}(r, 1)}{(1 - \tau)u_c(r, 1)} \tag{10}$$

$$\frac{\partial x_0}{\partial \tau} = \frac{x_0}{1 - \tau} \tag{11}$$

When the redistribution parameter is zero, everyone works; the "last" worker is the individual with lowest ability in the population.

6. The second-order condition for a maximum is $D \equiv x^2b^2u_{cc} - 2xbu_{cl} + u_{ll} - 2\alpha x^2 u_c < 0$, where $b \equiv 1 - \tau - 2\alpha xn = 1 - T'(y)$.

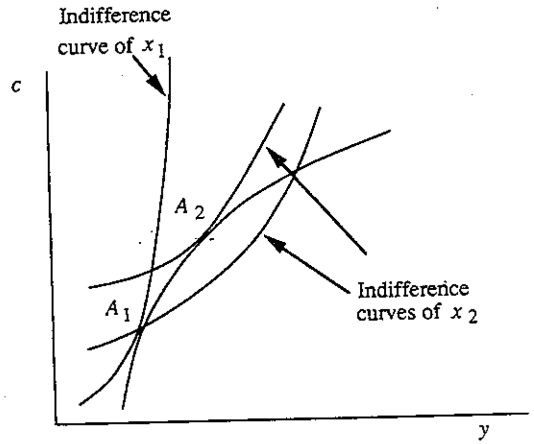

Figure 5-1.

To characterize the political equilibrium of the next section it is necessary to find conditions under which individuals with higher ability also have higher gross incomes or, to be more precise, the conditions under which the ordering of productivity corresponds to the ordering of earned income. The relation of the two orderings depends on the properties of the utility function.

It can be shown that the two orderings correspond if consumption and leisure are normal goods. For $\alpha \geqslant 0$ this is consequence of lemma 1 in Sadka (1976). Sadka shows that under the specified conditions the indifference curve (in the (c, y) plane) of a higher ability individual is less steep than that of a lower ability individual at *every* combination of consumption and of gross income.[7] His Figure 5-1 is reproduced (in our notation) as Figure 5-1 here. In the (c, y) plane all individuals face the same budget line which, from (5), is

$$c = r + (1 - \tau)y - \alpha y^2 \equiv y - T(y) \tag{12}$$

The restriction in (4) in conjunction with $\alpha \geqslant 0$ implies that the budget line has a positive and nonincreasing slope as drawn in Figure 5-1. Consider now two individuals with productivity levels x_1 and x_2 such that $x_2 > x_1$. The lower ability individual is in equilibrium at point A_1 where the budget line is tangent to his indifference curve. Since at every point, and at point A_1 in particular, the indifference curve of the higher ability individual is shallower, and since the budget line is positively sloped and concave, the tangency point of this individual occurs to the right of A_1, at a point such as A_2. Hence, for $\alpha \geqslant 0$, the rankings of abilities and of gross incomes are identical.

7. The difference in abilities shows up as a difference in tastes because of the focus on the individual's trade-off between consumption and gross income rather than on the trade-off between consumption and leisure for which all individuals possess the same indifference curves.

For $\alpha < 0$, normality of consumption is sufficient to establish that y and x are positively related. Performing a comparative statics experiment with respect to r on the first-order condition in (7), the resulting change in consumption is

$$\frac{dc}{dr} = \frac{1}{|D|}\{x[1 - T'(y)]u_{cl} - u_{ll} + 2\alpha x^2 u_c\}$$

Here D is the second-order condition for the problem in equation (6). Its explicit form appears in footnote 6. Performing a comparative statics experiment with respect to x and using this expression it can be shown that

$$\frac{dy}{dx} = \frac{1}{|D|}\left\{x[1 - T'(y)]u_c + n\left(|D|\frac{dc}{dr} - 2\alpha x^2 u_c\right)\right\}$$

Since $u_c > 0$, $\alpha < 0$, and consumption is a normal good $(dc/dr > 0)$, gross income and ability are positively related for the case $\alpha < 0$ also.

For $\alpha \geqslant 0$ the convexity of the individual's budget set assures that given the parameters r, τ, α, and x there is only one value of n that solves the maximization problem in equation (6). However, for $\alpha < 0$, the budget set is not convex so that multiple solutions for n may occur. We rule such situations out by requiring that the second partial derivative of (6) with respect to n (whose explicit form is in footnote 6) is negative for all $0 \leqslant x \leqslant x_u$ and $0 \leqslant n \leqslant 1$. This requirement constitutes a joint restriction on the degree of concavity of the utility function and on the degree of regressivity of the tax schedule when this schedule is regressive.

III. The Political Process and the Government Budget Constraint

Choice of the parameters of the tax function is a political economy decision. In our analysis, the parameters of the tax schedule are determined by majority voting. Therefore only tax schedules that cannot be defeated through majority rule by *any* other schedule can be a political equilibrium. Voters are informed about their tastes and about the effects of taxes and redistribution on earned income implied by the model in the previous section. Taxes are levied to finance a fixed level of per capita government expenditure, G, and the amount of lump sum per capita redistribution in the form of demogrants that results from the political process.

Let $F(x)$ denote the distribution function of individual productivity so that $F(x)$ is the fraction of the population with productivity not greater than x. Budgetary deficits and surpluses are not possible, so the budget is balanced by taxes on the incomes of those who work:

$$\int_{x0}^{x_u} \{\tau x n(x) + \alpha[xn(x)]^2\} dF(x) = G + r \tag{13}$$

Since G is taken to be exogenous it is set to zero for simplicity.[8] Equation (13) implicitly determines r as a function of τ and α. This function is denoted

$$r = r(\tau, \alpha) \tag{14}$$

Provided the utility function and the distribution function of individual ability are continuous, so is the function $r(\cdot)$. Following Romer (1975), we refer to (14) as the tax possibility frontier (TPF). Ignoring effects on incentives and income, an increase in either the flat component of the tax structure, τ, or in the degree of progressivity, α, increases revenues and the amount of per capita redistribution, r. When incentive effects are taken into account, an increase in either τ or α may decrease the tax base so much that redistribution must decrease to balance the budget. This is more likely if the tax burden is relatively high. The derivatives of r with respect to τ and α (denoted r_τ and r_α) may therefore be either positive or negative. The range along the TPF where either or both of r_τ and r_α are nonpositive is inefficient, since from any point in this range it is possible to increase redistribution without increasing tax burdens. Since *all* individuals like such changes, majority rule will never induce a political equilibrium along the inefficient range of the TPF. This is summarized in the following proposition.

Proposition 1: Under majority rule the voting equilibrium is never in the range of the TPF along which $r_\tau \leqslant 0$ or $r_\alpha \leqslant 0$, or both.

From equation (13), the derivatives of the TPF with respect to τ and α are[9]

$$r_\tau = \frac{1}{H}\int_{x_0}^{x_u}\left\{y(x) + xT'[y(x)]\frac{\partial n}{\partial \tau}(x)\right\} dF(x) \tag{15a}$$

$$r_\alpha = \frac{1}{H}\int_{x_0}^{x_u}\left\{[y(x)]^2 + xT'[y(x)]\frac{\partial n}{\partial \alpha}(x)\right\} dF(x) \tag{15b}$$

$$H \equiv 1 - \int_{x_0}^{x_u} xT'[y(x)]\frac{\partial n}{\partial r}(x) dF(x) \tag{15c}$$

where $\partial n/\partial z(x), z = \tau, \alpha, r$ are the responses of the labor supply of an individual with ability x to ceteris paribus changes in the tax parameters τ, α, and r. Assuming that a ceteris paribus increase in redistribution decreases individual labor supply (or does not increase it) $\partial n/\partial r \leqslant 0$. Since $T'[\cdot] \geqslant 0$, this implies that H is bounded away from zero and positive. Inspection of equations (15a) and (15b) reveals that, since $y(x)$, x, $\partial n/\partial \tau$, $\partial n/\partial \alpha$, and $T'[\cdot]$ are all bounded from above so are the numerators of these equations. Since H is bounded away from zero, this implies that r_τ and r_α are finite. Hence there is only one value of r that corresponds to each (τ, α) pair; the function $r(\tau, \alpha)$ is single valued.

8. The case in which there is an endogenously determined public good is briefly discussed in Section VIII.

9. The derivations are in part 1 of the appendix.

It is well known that majority rule induces collective intransitivities that can lead to cycles in the composition of the majority, when voters make multidimensional choices. This is an aspect of Arrow's (1951) impossibility theorem. In the contest of the present model, voters' preferences are affected by the three parameters r, τ, and α that define a tax schedule. The TPF in (14) reduces the dimensionality of the problem to the two parameters τ and α. But this alone does not ensure that there exists a unique tax schedule along the TPF that is preferred by a majority. In general there may be no Condorcet winner or, in the terminology of game theory, the core of the majority-rule game over tax schedules may be empty.

Proposition 1 implies that if there exists a majority-winning tax schedule, it must be in the range of the TPF in which $r_\tau > 0$ and $r_\alpha > 0$. The following discussion establishes conditions for the existence of a majority political equilibrium and characterizes it. Let $s \equiv (r, \tau, \alpha)$ be a tax schedule. Let S_m be the set of tax schedules along the TPF most preferred by the individual with median ability. This individual will be referred to as "the median individual" or simply "the median." Let s_m be a tax schedule in S_m. The set S_m may, but does not have to, include several tax schedules. Obviously when it does the median is indifferent between the different schedules. The following two definitions help to organize the discussion that follows.

Definition 1: A tax schedule s is a *local majority winner* if it is on the TPF and if there is no other schedule in the neighborhood of this schedule on the TPF that is strictly preferred by a majority.

Definition 2: A tax schedule is a *global majority winner* if it is on the TPF and if there is no other schedule on the TPF that is strictly preferred by a majority.

The local majority winner concept helps to characterize the conditions under which a global majority winner exists. In addition it clarifies the voters' choice when the political system contemplates only small changes in the status quo, as in Kramer and Klevorick (1974) and Romer (1977).

The following discussion, that culminates in theorem 1, demonstrates that all the schedules in the set S_m are local majority winners. Let

$$s_m \equiv (r_m, \tau_m, \alpha_m) \tag{16}$$

be any schedule in the set S_m. Let

$$dr \equiv r - r_m \qquad d\tau \equiv \tau - \tau_m \qquad d\alpha \equiv \alpha - \alpha_m \tag{17}$$

be a local deviation of s from s_m along the TPF. Let

$$I(r, \tau, \alpha; x) \equiv \max_n u[r + (1 - \tau)xn - \alpha(xn)^2, 1 - n] \tag{18}$$

be the indirect utility function of an individual with ability x. Since $u(\cdot)$ and the individual's budget constraint are continuous and the range of $u(\cdot)$ is compact, $I(\cdot)$ is a continuous function of the parameters r, τ, and α. In addition the implicit function theorem guarantees the differentiability of $I(\cdot)$ with respect to the three parameters.[10] Differentiating (18) totally with respect to a combined local change in the parameters of the tax schedule and using the envelope theorem

$$dI(y) = u_c(dr - y\,d\tau - y^2 d\alpha) \tag{19}$$

Lemma 1: The schedule s_m is a local majority winner whenever either of the following holds

$$\begin{array}{llll} \text{(i)} & d\tau > 0 & \text{and} & d\alpha > 0 \\ \text{(ii)} & d\tau > 0 & \text{and} & d\alpha = 0 \\ \text{(iii)} & d\tau = 0 & \text{and} & d\alpha > 0 \\ \text{(iv)} & d\tau < 0 & \text{and} & d\alpha < 0 \end{array}$$

Proof: Since $s_m \in S_m$, the median either dislikes or is indifferent to the change. Hence, from (19)

$$dI(y_m) = u_c^m(dr - y_m d\tau - y_m^2 d\alpha) \leqslant 0 \tag{20}$$

where the subscript or superscript designates that the appropriate quantity refers to the median. In case (i), since $d\tau > 0$ and $d\alpha > 0$

$$dI(y) < dI(y_m) \leqslant 0 \quad \text{for all } y > y_m \tag{21}$$

Since gross income is increasing in productivity, all the individuals with ability above that of the median dislike the change. If the median dislikes the change too there is a majority against it. If the median is indifferent there is a tie between those who like and those who dislike the change. In either case there is no schedule that is preferred to s_m by a majority. In cases (ii) and (iii) the inequality in (21) is still satisfied for all $y > y_m$. Hence the same considerations apply and there is no majority for the change.

In case (iv)

$$dI(y) < dI(y_m) \leqslant 0 \quad \text{for all } y < y_m \tag{22}$$

Hence if the median dislikes the change there is a majority against it, and if the median is indifferent the majority does not prefer the change. In either case s_m is a local majority winner for the set of changes specified in the lemma. □

Lemma 2: The schedule s_m is a local majority winner against any change in schedule such that $d\tau < 0$, $d\alpha > 0$.

[10] Varian (1978), p. 267.

Proof: The change in welfare experienced by an individual with income y as a result of the shift from s_m to s is given by equation (19). Since $u_c > 0$ for all y, the change in welfare is positive, negative, or zero depending on whether the following second-degree polynomial in y is positive, negative, or zero:

$$P(y) \equiv -(d\alpha)y^2 - (d\tau)y + dr \tag{23}$$

The roots of $P(y)$ are

$$y_{c1} = \frac{1}{2}\left[-\frac{d\tau}{d\alpha} - \sqrt{\left(\frac{d\tau}{d\alpha}\right)^2 + 4\frac{dr}{d\alpha}}\right] \tag{24a}$$

$$y_{c2} = \frac{1}{2}\left[-\frac{d\tau}{d\alpha} + \sqrt{\left(\frac{d\tau}{d\alpha}\right)^2 + 4\frac{dr}{d\alpha}}\right] \tag{24b}$$

Since $-d\alpha < 0$, the polynomial $P(y)$ has a maximum and looks like an inverted U. If both roots are imaginary, $P(y)$ is either positive or negative for all $y - s$. When $P(y_m) < 0$, this implies that $P(y) < 0$ for all y, and all voters dislike the change. If both roots are real and distinct (see panel a of Figure 5-2)

$$P(y) < 0 \quad \text{for all } y > y_{c2} \text{ and } y < y_{c1} \tag{25a}$$

$$P(y) > 0 \quad \text{for all } y_{c1} < y < y_{c2} \tag{25b}$$

$$P(y) = 0 \quad \text{for } y = y_{c1} \text{ and } y = y_{c2} \tag{25c}$$

By assumption $P(y_m) < 0$, so y_m must be in the range defined by equation (25a). If $y_m > y_{c2}$, at least all individuals with incomes above y_m dislike the change too, and if $y_m < y_{c1}$, at least all individuals with incomes below y_m dislike the change. In either case there is a majority against the change.

When $P(y_m) = 0$, y_m is equal to either y_{c1} or y_{c2}. If y_{c1} and y_{c2} are distinct, the previous argument implies that there is always a majority against the change. If y_{c1} and y_{c2} collapse to one root, y_m equals its common value and $P(y)$ touches the horizontal axis only at this point. All other points are therefore below the horizontal axis, since $P(y)$ has a maximum at y_m. Hence everybody except for the median strictly dislikes the change. It follows that s_m is a local majority winner for all changes of the type $d\tau < 0$ and $d\alpha > 0$. □

Lemma 3: The schedule s_m is a local majority winner against any change in schedule such that

$$dr \leqslant 0 \qquad d\tau > 0 \qquad d\alpha < 0$$

Proof: When $d\alpha < 0$, $P(y)$ has a minimum and looks like a U (panel b of Figure 5-2). The value of $P(y)$ at the minimum is

$$\frac{1}{4}\frac{(d\tau)^2}{d\alpha} + dr \tag{26}$$

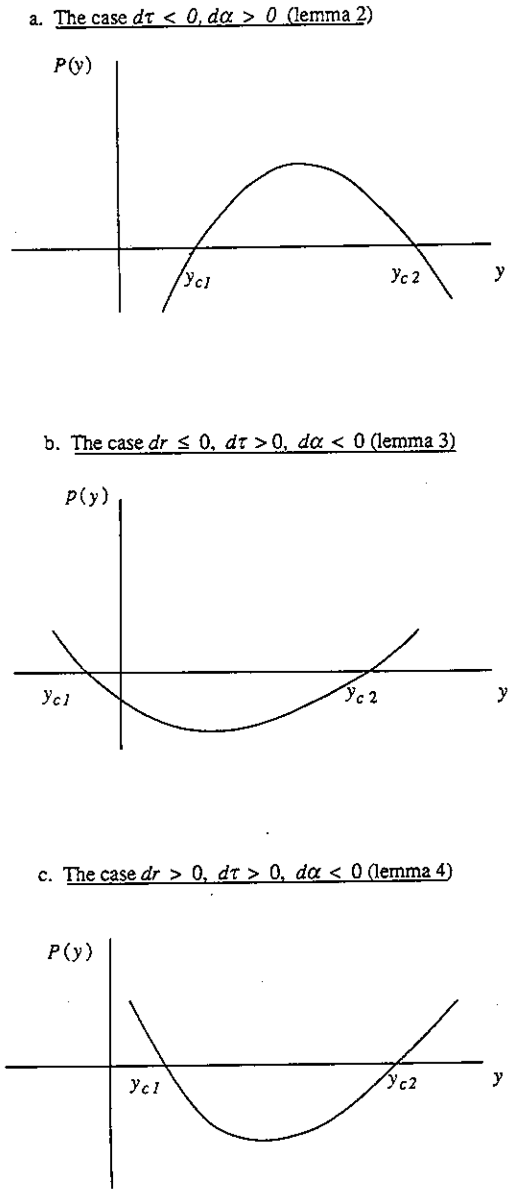

***Figure* 5-2.**

which is negative, since $d\alpha < 0$ and $dr \leqslant 0$. Since for extreme values of y, $P(y)$ eventually becomes positive, the two roots—y_{c1} and y_{c2}—are real and distinct.

Since $dr \leqslant 0$ and $d\alpha < 0$ (24a) implies that $y_{c1} \leqslant 0$. Since $P(y_m) \leqslant 0$, $y_{c1} \leqslant y_m \leqslant y_{c2}$. Hence at least all the individuals with incomes below y_m weakly dislike the change. Since the median also weakly dislikes the change there is no majority in favor of the change and s_m is a local majority winner. □

From the TPF in (14)

$$dr = r_\tau d\tau + r_\alpha d\alpha \tag{27}$$

Dividing (27) by $d\alpha$ and substituting it into equations (24a) and (24b)

$$y_{c1}(a) = \frac{1}{2}(a - \sqrt{a^2 - 4r_\tau a + 4r_\alpha}) \tag{28a}$$

$$y_{c2}(a) = \frac{1}{2}(a + \sqrt{a^2 - 4r_\tau a + 4r_\alpha}) \tag{28b}$$

$$a \equiv -\frac{d\tau}{d\alpha} \tag{28c}$$

The only type of change in schedules not covered by lemmas 1 through 3 is the case $dr > 0$, $d\tau > 0$, $d\alpha < 0$. The next lemma demonstrates that if there is an internal solution along the TPF for s_m, this case can be disregarded.

***Lemma 4*:** If the schedule s_m preferred by the median individual does not occur along the boundary[11] of the TPF, changes of the type $dr > 0$, $d\tau > 0$, $d\alpha < 0$ cannot occur.

Proof: Since $d\alpha < 0$ and $dr > 0$, $dr/d\alpha < 0$. It follows from equation (24a) that $y_{c1}(a) > 0$ for all a's in the range defined by the conditions of the lemma. Since $d\alpha < 0$, $P(y)$ has a minimum and can therefore be drawn as in panel c of Figure 5-2. Equation (27) implies that $dr > 0$ is equivalent to $a > r_\alpha/r_\tau$. Since $P(y_m) \leqslant 0$, $y_m < y_{c2}(a)$. Rearranging, this is equivalent to

$$a \leqslant \frac{r_\alpha - y_m^2}{r_\tau - y_m}$$

Hence we need to examine only cases for which a is in the range

$$\frac{r_\alpha}{r_\tau} < a \leqslant \frac{r_\alpha - y_m^2}{r_\tau - y_m} \tag{29}$$

11. The boundary of the TPF is defined by the restrictions $r > 0$ and $-\tau/2x_u \leqslant \alpha \leqslant (1 - \tau)/2x_u$. Hence the boundary is attained when $r = 0$ or $\alpha = -\tau/2x_u$ or $\alpha = (1 - \tau)/2x_u$.

In all other cases either dr is not positive or the median likes the change, contradicting the fact that s_m is weakly preferred by the median to all other schedules.

Since s_m occurs at an internal point along the TPF it must satisfy (applying the envelope theorem to (18)) the following two first-order conditions

$$\frac{dI_m}{d\alpha} = -u_c^m(\cdot)y_m\left(\frac{d\tau}{d\alpha} + y_m\right) = 0 \tag{30a}$$

$$\frac{dI_m}{dr} = u_c^m(\cdot)\left(1 - y_m\frac{d\tau}{dr}\right) = 0 \tag{30b}$$

where $d\tau/d\alpha$ is the change in τ resulting from a change in α along the TPF for a given r, and $d\tau/dr$ is the change in τ resulting from a change in r along the TPF holding α constant. For changing τ and α and constant r, equation (27) implies

$$\frac{d\tau}{d\alpha} = -\frac{r_\alpha}{r_\tau} \tag{31}$$

If the median works, $u_c^m(\cdot)y_m > 0$ and equations (30a) and (31) imply

$$y_m = \frac{r_\alpha}{r_\tau} \tag{32}$$

Substituting (32) into the upper limit of the interval in (29)

$$\frac{r_\alpha - y_m^2}{r_\tau - y_m} = \frac{r_\alpha - (r_\alpha/r_\tau)^2}{r_\tau - (r_\alpha/r_\tau)} = \frac{r_\alpha r_\tau^2 - r_\alpha^2}{r_\tau^3 - r_\alpha r_\tau} = \frac{r_\alpha}{r_\tau}$$

Hence the interval in (29) reduces to

$$\frac{r_\alpha}{r_\tau} < a \leqslant \frac{r_\alpha}{r_\tau} \tag{33}$$

implying that there is no value of a that simultaneously satisfies $dr > 0$ and $P(y_m) \leqslant 0$. Hence changes of the type $dr > 0$, $d\tau > 0$, $d\alpha < 0$ can be disregarded.

If the median does not work, taxes are set to yield the maximum redistribution for the TPF; s_m is characterized by the two first-order conditions $r_\alpha = r_\tau = 0$. It follows from equation (27) that $dr = 0$ for all $d\alpha$ and $d\tau$ when the median does not work. Hence changes of the type $dr > 0$ are not possible. □

Theorem 1: Let S_m be the set of tax schedules along the TPF such that there is no other tax schedule along the TPF that is strictly preferred by the individual with median ability. Let s_m be an element of S_m. If s_m does not occur on the boundary of the TPF, then s_m is a local majority winner.[12]

Proof: The proof is a direct consequence of lemmas 1 through 4.

Suppose that a given schedule, s_1, along the TPF is the status quo. Now an alternative schedule, s_2, is proposed. If s_2 is strictly preferred by a majority, it becomes the status quo schedule. In all other cases, including ties, the original status quo s_1 remains in place unless another schedule that is strictly preferred by a majority is found. The process ends when no schedule that is strictly preferred by a majority to the status quo can be found. Using this mechanism to vote on tax schedules, theorem 1 implies that, once a schedule $s_m \in S_m$ becomes the status quo, the process stops and s_m is adopted provided only that schedules in the neighborhood of the status quo are considered as alternatives at each stage. If S_m is not a singleton, this does not pin down which of the schedules in S_m will be adopted. But it limits the search for equilibrium schedules to the set that is most preferred by the individual with median ability. This characteristic of the local majority-winning set of schedules proves useful for the characterization of the equilibrium degree of progressivity in Section V.

IV. Conditions for the existence of a Globally Winning Tax Schedule

The result of theorem 1 can be extended to any (global) change in schedule along the TPF provided some additional conditions are satisfied. The following discussion leads to a statement of the precise conditions. Let

$$n(s, x),\ y(s, x)$$

be the labor input and the gross labor income of an individual with productivity x when the tax schedule is s. Let

$$\Delta r \equiv r - r_m \qquad \Delta\tau \equiv \tau - \tau_m \qquad \Delta\alpha \equiv \alpha - \alpha_m \tag{34}$$

be any global movement away from s_m along the TPF. The following lemma establishes that if an individual dislikes this change, his net income or consumption, with his employment fixed at $n(s_m, x)$, must be lower after the change.

12. The set S_m does not always contain all the local majority winning tax schedules. For example there may exist a tax schedule, s_l, along the TPF and not in S_m that is strictly preferred by the median individual to any schedule in the vicinity of s_l. This occurs when none of the schedules in S_m is in the vicinity of s_l. Since s_l does not belong to S_m, the median individual strictly prefers any of the schedules in S_m to s_l. Nonetheless s_l is a local majority winner, since the arguments leading to theorem 1 imply that if the median individual weakly prefers s_l to any schedule in the neighborhood of s_l, there is a weak majority in favor of s_l whenever s_l is confronted with any of the schedules in the vicinity of s_l. Hence the set of local majority winners may be larger than S_m. But (subject to the conditions of Section IV below) the set of global majority winners is identical to S_m since any of the (more distant) schedules in S_m is weakly preferred by a majority to s_l.

Lemma 5: If an individual with productivity x dislikes (or is indifferent to) the shift from $s_m = (r_m, \tau_m, \alpha_m)$ to $s = (r, \tau, \alpha)$, then, holding his labor input constant at $n(s_m, x)$, his net income or consumption is lower at s than at s_m.

Proof: Given that the individual's employment remains at $n(s_m, x)$, the change in his consumption is, from equation (5)

$$-\Delta\alpha[y(s_m, x)]^2 - \Delta\tau y(s_m, x) + \Delta r \tag{35}$$

Suppose the expression in (35) is nonnegative, contrary to the assertion of the lemma. Since leisure remains the same, the welfare of the individual obviously increases if (35) is positive. When the individual is allowed to adjust his labor input, welfare increases even further, since he is allowed to take advantage of additional substitution possibilities between labor and leisure that were not available when labor was frozen as $n(s_m, x)$. If the expression in (35) is zero, leisure and consumption are unchanged, so his welfare does not change. But once he is allowed to adjust his labor, welfare increases. Hence if the expression in (35) is nonnegative the individual's welfare increases as a result of the change, contradicting the assumption that the individual dislikes the change. It follows that given that labor is frozen at $n(s_m, x)$, consumption is lower at s than at s_m if the individual dislikes the change. □

An important consequence of lemma 5 is that if the median dislikes the switch from s_m to s, his net income at the prechange level of work must be reduced by the change.

Theorem 2 below presents sufficient conditions for the existence of a global majority winner (s_m) and characterizes the conditions. The statement of the theorem requires some preliminary discussion, to which we turn next. Let $P^g(y)$ be the polynomial in (23) with the local changes (dr, $d\tau$, $d\alpha$) replaced everywhere by global changes—(Δr, $\Delta\tau$, $\Delta\alpha$). Let

$$\bar{r}_\tau \equiv \frac{r(\tau, \alpha_m) - r(\tau_m, \alpha_m)}{\tau - \tau_m} \tag{36a}$$

$$\bar{r}_\alpha \equiv \frac{r(\tau, \alpha) - r(\tau, \alpha_m)}{\alpha - \alpha_m} \tag{36b}$$

Then by definition

$$\Delta r = \bar{r}_\tau \Delta\tau + \bar{r}_\alpha \Delta\alpha \tag{37}$$

Let y^g_{c1} and y^g_{c2} be the roots of the polynomial $P^g(y)$. The explicit form of those roots is the same as in equations (24), after adjusting for the change from local to the global changes denoted by "Δ." In the case $\Delta\alpha < 0$, the polynomial has a minimum and is described qualitatively by either panel b or c of Figure 5-2.

Dividing (37) by $\Delta\alpha$ and substituting the result into the expressions for y^g_{c1} and y^g_{c2} we obtain

$$y^g_{c1}(b) = \frac{1}{2}(b - \sqrt{b^2 - 4\bar{r}_\tau b + 4\bar{r}\alpha}) \tag{38a}$$

$$y^g_{c2}(b) = \frac{1}{2}(b + \sqrt{b^2 - 4\bar{r}_\tau b + 4\bar{r}\alpha}) \tag{38b}$$

$$b \equiv -\frac{\Delta\tau}{\Delta\alpha} \tag{38c}$$

At prechange labor inputs, all individuals with gross labor incomes in the range

$$\max[0, y_{c1}] < y < y_{c2} \tag{39}$$

experience a drop in net income as a result of the change. But those who are just a little above $\max[0, y_{c1}]$ or just a little below y_{c2} may actually experience a small increase in welfare (in spite of the income drop) once they adjust their labor inputs optimally. Hence the set of prechange gross incomes at which welfare decreases as a result of the change is strictly contained in the set defined by (39). Condition (i) of theorem 2 below and the fact that $P^g(y)$ decreases as one moves away from y^g_{c1} and y^g_{c2} towards $b/2$ imply that the smaller set is convex. Hence there exist small but positive numbers $\varepsilon_1(a)$ and $\varepsilon_2(a)$ such that welfare decreases for all individuals with prechange gross income in the range

$$\max[0, y^g_{c1}(b) + \varepsilon_1(b)] < y < y^g_{c2}(b) - \varepsilon_2(b) \tag{40}$$

Given a tax schedule, an individual's level of labor input and therefore his gross income is determined by his productivity, x. Hence gross income of an individual with productivity x when the tax schedule is s_m can be written

$$y = y(s_m, x) \tag{41}$$

Let

$$x = \hat{x}(s_m, y) \equiv x\{y\} \tag{42}$$

be the inverse function to (41). Given s_m, it expresses x as a function of y. Since, given the tax schedule, the ranking of individuals by productivity coincides with their ranking by gross income, $x(y)$ is an increasing function of y.

Theorem 2: Let $\Delta r \equiv r - r_m$, $\Delta\tau \equiv \tau - \tau_m$, $\Delta\alpha \equiv \alpha - \alpha_m$ be any global change in the tax schedule along the TPF starting from a status quo at $s_m = (r_m, \tau_m, \alpha_m)$. Let the conditions of theorem 1 be satisfied. Then s_m is a global majority winner provided the following additional conditions are satisfied.

(i) If individual i dislikes the change, all individuals whose net income decreases by more than that of individual i, when employment is held fixed at prechange levels, dislike the change also.

(ii)

$$F\{x[y^g_{c2}(b)-\varepsilon_2(b)]\}-F(x\{\max[0,\ y^g_{c1}(b)+\varepsilon_1(b)]\})\geqslant\frac{1}{2} \tag{43}$$

for all b's that correspond to changes of the type $\Delta r \leqslant 0$, $\Delta\tau > 0$, $\Delta\alpha < 0$.

(iii)

$$F\{x[y^g_{c2}(b) - \varepsilon_2(b)]\} - F\{x[y^g_{c1}(b) + \varepsilon_1(b)]\} \geqslant \frac{1}{2} \tag{44}$$

for all b's that satisfy

$$\frac{\bar{r}_\alpha}{\bar{r}_\tau} < b < \frac{r_\tau^2 - (r_\alpha/\bar{r}_\alpha)r_\alpha}{r_\tau^2 - (r_\tau/\bar{r}_\tau)r_\alpha}\frac{\bar{r}_\alpha}{\bar{r}_\tau} \tag{45}$$

and that correspond to changes of the type $\Delta r > 0$, $\Delta\tau > 0$, $\Delta\alpha < 0$.

Proof: In part 2 of the appendix.

Theorem 2 implies that every $s_m \in S_m$ is a global majority winner. In addition, since any tax schedule on the TPF but not in S_m is not strictly preferred by a majority, the set S_m contains *all* the majority-winning schedules.[13]

Some discussion of the conditions in theorem 2 is in order. Condition (i) is basically an implicit restriction on the possible set of utility functions. Condition (ii) is always satisfied when $y^g_{c1}(b) + \varepsilon_1(b) \leqslant 0$, since $0 \leqslant y_m \leqslant y^g_{c2}(b) - \varepsilon_2(b)$. The reason is that in all such cases at least half of the voters do not strictly prefer the change. This is more likely to be the case when $\varepsilon_1(b)$ is small. In turn, this will be the case when the utility difference between a compensating change in income that maintains the same level of net income and a compensating change that maintains the same level of utility is small. Condition (iii) needs to be satisfied only in a rather limited range. This condition requires that the density of individuals in the range of abilities around the median is sufficiently large. In more intuitive but less exact terms, it requires that the middle class be a sufficiently large fraction of society. Note that when $r_\alpha/\bar{r}_\alpha = r_\tau/\bar{r}_\tau$, the range of b over which this condition is required to hold is empty, so that the condition is not binding.[14] When $r_\alpha/\bar{r}_\alpha \neq r_\tau/\bar{r}_\tau$, the range in (45) is narrower (and the condition in (44) therefore less restrictive) the nearer are the ratios $r_\alpha/\bar{r}_\alpha$ and $r_\tau/\bar{r}_\tau$ to each other. Those ratios tend to be similar when the differences between the

13. In game theoretic terms this means that S_m and the core of the majority-rule game over tax schedules coincide.

14. In particular this condition is satisfied when $r_\alpha = \bar{r}_\alpha$ and $r_\tau = \bar{r}_\tau$ so that the TPF is a plane in the range between (τ_m, α_m) and (τ, α).

global and the local curvatures of the TPF in the τ and the α directions are of similar magnitudes.

V. Majority Rule and Income Tax Progressivity when the Decisive Voter Works

The previous two sections have established the decisiveness of the individual with median ability for the choice of tax schedule. Hence the choice of schedule under majority rule depends on the position of the median (or decisive) voter in the income distribution, and therefore on his level of ability or productivity. This section establishes conditions under which a decisive voter who works chooses to tax incomes at marginally progressive rates. The following section analyzes the tax schedule chosen by a voter who subsists on transfer payments and does not work.

The decisive voter's problem is to choose the parameters of the TPF by maximizing his indirect utility (18)

$$I[r, \tau, \alpha; x_m] \tag{46}$$

subject to the TPF in (14). Here x_m is the productivity of the person at the median of the ability distribution. Substituting (14) into (46), differentiating totally with respect to α and r, and using the envelope theorem, we obtain

$$\frac{dI_m}{d\alpha} = u_c(r_\alpha - y_m^2) \tag{47a}$$

$$\frac{dI_m}{d\tau} = u_c(r_\tau - y_m) \tag{47b}$$

The subscript m denotes the voter with median ability and income, so y_m is the pretax income of the decisive voter and I_m is his (indirect) utility. We focus on the case in which the tax schedule that is preferred by the median is not on the boundary of the TPF. In this case the parameters of s_m are obtained by equating equations (47a) and (47b) to zero. The resulting equations together with the TPF constitute three equations that, in principle, can be used to determine the three parameters r_m, τ_m, and α_m.[15]

We are less interested in explicit solutions for the three parameters than in the conditions under which majority rule results in rising marginal tax rates. Earlier studies that used a political economy framework, by Romer (1975), Roberts (1977), and Meltzer and Richard (1981, 1983), ruled out marginal progressivity by imposing a linear tax function. The tax function used here can accommodate either marginal progressivity ($\alpha > 0$), marginal regressivity

15. Similarly, corner solutions can be obtained by incorporating the constraints $r \geqslant 0$ and $-\tau/2x_u \leqslant \alpha \leqslant (1 - \tau)/2x_u$.

($\alpha < 0$), or a constant marginal tax rate ($\alpha = 0$). We can derive a sufficient condition for marginal progressivity by requiring that the expression in (47a) be positive for all $\alpha \leqslant 0$. Using (15b) in (47a) and noting that $u_c > 0$, this is equivalent to the condition

$$\frac{\int_{x_0}^{x_u}\{[y(x, r, \alpha)]^2 + xT'[y(x, r, \alpha)]\partial n/\partial\alpha(x, r, \alpha)\}dF(x)}{1 - \int_{x_0}^{x_u} xT'[y(x, r, \alpha)]\partial n/\partial r(x, r, \alpha)dF(x)} > [y(x_m, r, \alpha)]^2 \quad \text{for all } \alpha \leqslant 0 \text{ and } \tau \qquad (48)$$

The dependence of the various terms in (48) on the productivity class x and on the parameters r and α is recognized explicitly in the notation. By contrast, the parameter τ that is held constant in (47a), and therefore in (48), is subsumed into the functional forms.

Equation (48) assures that *all* (of the possibly many) equilibria are in the progressive range ($\alpha > 0$). The reason is that increases in α increase utility for *all* $\alpha \leqslant 0$, so the decisive voter will never stop at a nonpositive value of α. Using basic formulas for the mean and the variance and rearranging, the condition in (48) can be restated as

$$\begin{aligned} C(r,\alpha) \equiv {} & [1 - F(x_0)]\{V(r,\alpha) + [\bar{y}(r,\alpha)]^2\} - [y(x_m, r,\alpha)]^2 \\ & + \int_{x_0}^{x_u} xT'[y(x,r,\alpha)]\left\{\frac{\partial n}{\partial\alpha}(x,r,\alpha) + [y(x_m, r,\alpha)]^2 \right. \\ & \left. \times \frac{\partial n}{\partial r}(x,r,\alpha)\right\} dF(x) > 0 \quad \text{for all } \alpha \leqslant 0 \text{ and } \tau \qquad (49) \end{aligned}$$

Here $\bar{y}(\cdot)$ and $V(\cdot)$ are the mean and the variance of income of the working population. At $\alpha = 0$ (a linear tax schedule), condition (49) reduces to

$$\begin{aligned} C(r,0) = {} & [1 - F(x_0)]\{V(r,0) + [\bar{y}(r,0)]^2\} - [y(x_m, r,0)]^2 \\ & + \tau\int_{x_0}^{x_u} x\left\{\frac{\partial n}{\partial\alpha}(x,r,0) + [y(x_m, r,0)]^2\frac{\partial n}{\partial r}(x,r,0)\right\}dF(x) > 0 \quad \text{for all } \tau \qquad (49a) \end{aligned}$$

Other things equal, conditions (49) and (49a) are more likely to be satisfied the larger the variance of income $V(\cdot)$ and the larger the mean income $\bar{y}(\cdot)$ in relation to median income $y(x_m, r, \alpha)$. Given α and r, $V(\cdot)$ is normally larger the larger the variance of abilities. The mean-median income spread is normally larger the larger is the degree of positive skewness in the distribution of abilities. This leads to the following proposition.

Proposition 2: Majority rule is more likely to produce a progressive tax structure the more spread out is the distribution of abilities and the larger the degree of positive skewness of this distribution.

Proposition 2 implies that, other things equal, voters are more likely to choose a progressive tax structure the larger the spread in the distribution of abilities and the more the distribution is skewed toward high productivity. The intuition underlying these results is familiar. Individuals with higher ability have a larger share of income than of votes. They earn higher incomes because their productivity is higher and, often, because they work more hours. Even if they work fewer hours, income and productivity are positively related in our model. Since the number of people with high productivity is relatively small, the median voter can readily form a majority that agrees to lower the flat tax rate τ and raise the degree of progressivity (or lower regressivity). The median voter chooses to do this if he can reduce his own tax burden without reducing the transfers he, and all others, receive from the budget.

A larger spread in the distribution of income increases the likelihood of marginal progressivity. The reason is that a wide spread of the distribution increases the tax base at relatively high incomes. The larger tax base increases the likelihood that a compensated increase in α will generate sufficient additional tax revenues to reduce the average tax paid by the decisive voter.

Effects on incentives modify the results just discussed. Income taxes alter labor-leisure choices and change the tax base. Some notion about the effects of incentives can be obtained by examining the conditions for $\alpha > 0$ when the first-order condition in (47b) for an optimal choice of τ is satisfied. Since $u_c > 0$, this condition implies $y_m = r_\tau$. Hence a sufficient condition for $\alpha > 0$ is, from (47a),

$$r_\alpha - r_\tau^2 > 0 \quad \text{for all } \alpha \leqslant 0 \qquad (50)$$

Using equations (15) in (50), noting that $H > 0$, and rearranging, this is equivalent to

$$\int_{x_0}^{x_u}\left(y^2 + xT'\frac{\partial n}{\partial\alpha}\right)dF\left(1 - \int_{x_0}^{x_u} xT'\frac{\partial n}{\partial r}dF\right) - \left[\int_{x_0}^{x_u}\left(y + xT'\frac{\partial n}{\partial\tau}\right)dF\right]^2 > 0 \quad \text{for all } \alpha \leqslant 0 \qquad (51)$$

where the dependence on x and s has been suppressed for notational simplicity. Since $\partial n/\partial r < 0$

$$H \equiv 1 - \int_{x_0}^{x_u} xT'\frac{\partial n}{\partial r}dF > 0$$

Since r_α and r_τ are positive this implies that

$$\int_{x_0}^{x_u}\left(y^2 + xT'\frac{\partial n}{\partial\alpha}\right)dF \qquad \int_{x_0}^{x_u}\left(y + xT'\frac{\partial n}{\partial\tau}\right)dF$$

are both positive. Since $\partial n/\partial\alpha$ and $\partial n/\partial\tau$ are both negative, this implies that a positive α is more likely the lower $|\partial n/\partial\alpha|$ and the higher $|\partial n/\partial\tau|$ and $|\partial n/\partial r|$ in the range $\alpha \leqslant 0$. This is summarized in the following proposition.

Proposition 3: Majority rule is more likely to produce a progressive tax structure the lower the disincentive effects of an increase in α and the higher the disincentive effects of an increase in either τ or r on employment.

The intuition underlying proposition 3 is the following. An increase in α can be used either to increase redistribution, r, or to decrease the flat component of the tax structure, τ, or to do a bit of both. To understand the intuition it is convenient to consider pure cases in which the increase in α is used either to reduce τ or to increase r. In the first case the tax burden on low-ability individuals is alleviated and the tax burden on high-ability individuals is increased. This is a good strategy for the median when the tax base is not decreased by much. This is the case if employment is relatively insensitive to the increase in α ($|\partial n/\partial \alpha|$ small) and relatively sensitive to the increase in τ ($|\partial n/\partial \tau|$ large). This will be the case, in turn, if the labor supply of high-income individuals is less sensitive to a change in the marginal rate of taxation than the labor supply of low-income individuals.

Consider now an increase in α that is used solely to increase redistribution r. As can be seen from equations (15a) and (15b), large values of $|\partial n/\partial r|$ reduce both r_α and r_τ because of the larger negative incentive effects of an increase in redistribution on work. But, as can be seen from (50), the median finds it worthwhile to increase α whenever r_α is larger than r_τ^2. Since $H > 1$, larger $|\partial n/\partial r|$ reduce r_τ^2 by more than they reduce r_α. The incentive to increase α is therefore larger when $|\partial n/\partial r|$ is larger.

Condition (49) is sufficient for global progressivity whereas condition (49a) assures progressivity only locally. Since condition (49a) is simpler it is useful to know under what circumstances this condition alone is sufficient for global progressivity. If $C(r, \alpha)$ is a decreasing function of α, condition (49a) alone is sufficient. The reason is that, for negative α's, $C(\cdot)$ is a fortiori positive. We turn therefore to a discussion of the channels through which an increase in α affects $C(\cdot)$.

An increase in α corresponds to a movement toward higher values of α and r along the TPF, keeping τ constant. The increase in α produces (at the original levels of income) an increase in the tax burdens of all working individuals so that they reduce their levels of work. The increase in r causes a further reduction in the labor input of all individuals. If $\partial n/\partial \alpha$ and $\partial n/\partial r$ do not differ much across individuals, the work levels of different individuals decrease by roughly similar amounts, but the incomes of abler individuals decrease by more. As a consequence, a movement toward higher values of α and r along the TPF produces a decrease in both the mean and the variance of income. The increase in α increases the tax burden of individuals with higher incomes relative to taxpayers as a group. If, as a result, $|\partial n/\partial \alpha|$ is larger for individuals with larger incomes, the downward effect on V and $\bar{y}$ is even stronger. The increase in r also raises the threshold productivity level below which individuals choose to remain idle. This raises x_0 and lowers $1 - F(x_0)$.

On the other hand, the increase in α and r by decreasing $y(x_m, r, \alpha)$ and by raising the lower limit of the integral in (49) tends to increase $C(\cdot)$ as α and r increase along the TPF. If the elasticity of individual labor supply with respect

to the combined change in α and r is smaller than 1 in absolute value, $T'[\cdot]$ increases,[16] countering some of the effects just described by increasing the weights on the negative $\partial n/\partial \alpha$ terms. In sum, for negative values of α, when the negative effects through V, $\bar{y}$, $1 - F(x_0)$, and possibly $T'[\cdot]$ dominate any positive effects,

$$\frac{\partial C(r, \alpha)}{\partial \alpha} < 0 \quad \text{for } \alpha < 0 \tag{52}$$

When the condition in (52) is satisfied, (49a) is sufficient to ensure that $\alpha > 0$.

VI. Majority Rule and Tax Progressivity when the Decisive Voter Does Not Work

In most democracies, the decisive voter works. He may receive transfers, but he also pays taxes. His decision to impose marginal progressivity depends on the balancing of the gains and losses he experiences and thus on parameters of the distribution of income and the labor supply response of those who pay the highest marginal rates. A nonworker faces a simpler problem. He pays no taxes, so his interest in marginal progressivity is greater. His own welfare depends on the transfers he receives, but these transfers do not increase with marginal progressivity if disincentive effects on income earners are strong. Consequently, a rational decisive voter who does not work never chooses a value of α that lowers redistribution.

The utility of a decisive voter who does not work increases monotically with r. To maximize utility, he chooses the point on the TPF at which r is maximized. Formally, we can state his problem as

$$\max_{\tau, \alpha} r(\tau, \alpha) \tag{53}$$

The solution to this problem yields the following two first-order conditions.

$$r_\alpha(\tau, \alpha) = 0 \tag{54a}$$

$$r_\tau(\tau, \alpha) = 0 \tag{54b}$$

from which, together with (14), it is possible to solve in principle for the decisive voter's preferred tax schedule s_m. Using (15b) and the fact that H is positive, a sufficient condition for progressivity is

$$\int_{x_0}^{x_u} \left(y^2 + xT' \frac{\partial n}{\partial \tau}\right) dF > 0 \quad \text{for all } \alpha \leqslant 0 \text{ and } \tau \tag{55}$$

[16]The total change in $T'[\cdot]$ at income y is $2y(1 + \eta_{n\alpha}^T)$ where $\eta_{n\alpha}^T$ is the (negative) elasticity of labor supply with respect to the combined increase in α and r along the TPF. Hence if $|\eta_{n\alpha}^T| < 1$, $T'[\cdot]$ increases as a result of the change.

which is equivalent to

$$K(r,\alpha) \equiv [1 - F(x_0)](V + \bar{y}^2) - \int_{x_0}^{x_u} xT' |\frac{\partial n}{\partial \alpha}| dF > 0 \quad \text{for all } \alpha \leqslant 0 \text{ and } \tau. \tag{56}$$

At $\alpha = 0$ this condition reduces to

$$[1 - F(x_0)](V + \bar{y}^2) - \tau \int_{x_0}^{x_u} x \left|\frac{\partial n}{\partial \alpha}\right| dF > 0 \quad \text{for all } \tau \tag{56a}$$

We saw in the previous section that when, given τ, we move in the direction of a higher α and a higher r along the TPF $(1 - F(x_0))(V + \bar{y}^2)$ goes down. Hence if the last term in (56) does not go down, $K(r(\alpha, \tau), \alpha)$ is (given τ) a decreasing function of α. This leads to the following proposition.

Proposition 4: If the median does not work and $A(r, \alpha) \equiv \int_{x_0}^{x_u} xT' |\partial n/\partial \alpha| dF$ is a nondecreasing function of α for all $0 \leqslant \tau \leqslant 1$, condition (56a) is sufficient to ensure marginal progressivity.

$A(r, \alpha)$ will be nondecreasing in α if $|\eta^T_{n\alpha}| < 1$ for all x and if the reduction in $A(\cdot)$ due to the increase in x_0 does not dominate the increase in $A(\cdot)$ because of the increase in T'. The previous statement implicitly assumes that $|\partial n/\partial \alpha|$ does not depend on α. If $|\partial n/\partial \alpha|$ increases with α (implying that the marginal disincentive effect of α on work grows as the tax burden increases), $A(\cdot)$ is more likely to increase with α.

Proposition 4 confirms the intuition that some of the factors that induce marginal progressivity are the same whether or not the median works. In particular, the larger the variance of abilities, the more skewed to the right is their distribution; and the smaller the disincentive effects of an increase in α, the more likely that majority rule will produce a marginally progressive tax schedule.

Finally, comparison of conditions (49) and (49a) with conditions (56) and (56a), respectively, suggests that if a progressive tax schedule arises when the median works it must arise a fortiori when the median does not work.

VII. A Remark on Progressivity in the Presence of Public Good

Since individual utility from expenditures on public goods has not been modeled explicitly, one may get the erroneous impression that progressivity of the tax schedule arises only when the budget is used for redistributional purposes. In fact the same elements that are conducive to progressivity when the budget is used to redistribute income are likely to lead to progressivity when it is used to provide a public good. A general demonstration of this claim is beyond the scope of this chapter. Instead we illustrate it for a particular utility function in the case in which the entire budget is used to finance a public good.

Let g be the amount of a public good enjoyed by a representative individual and let

$$v(c + g, l) \tag{57}$$

be the utility function of a typical individual. This specification, which implies that c and g are perfect substitutes, is adopted for simplicity. The TPF in (13) is replaced by

$$g = \delta N \int_{x_0}^{x_u} \{\tau x n(x) + \alpha [x n(x)]^2\} dF(x) \equiv \delta N r \tag{13a}$$

where N is the number of individuals in the economy and δ is a parameter between $1/N$ and 1. When $\delta = 1$, g is a pure public good. When $\delta = 1/N$, g is a publicly provided private good. In the more likely intermediate cases, $1/N < \delta < 1$, g is a public good but not a pure public good. Private consumption is now given by

$$c(x) = (1 - \tau)xn - \alpha(xn)^2 \tag{5a}$$

For $\delta = 1/N$, equation (13a) implies that $g = r$, so the model of this section becomes formally identical to the model of the previous sections. Hence the same factors that were conducive to progressivity before are conducive to progressivity now as well.

For $\delta > 1/N$, g is larger than r by some fixed factor. As a result, all individuals prefer a larger budget than in the case $\delta = 1/N$. The reason is that the marginal utility of the public good is now higher. However, the same conflicts of interest regarding the *financing* of g that existed when $\delta = 1/N$ are also present when $\delta > 1/N$. In particular the analysis of Sections III through VI can be replicated with r replaced by $\delta N r$. The formal conditions for the existence of a majority-winning schedule and for progressivity have to be adjusted to reflect this change. However, the qualitative results of propositions 2 and 3 are likely to carry over to this case too. The intuitive reasoning underlying this statement relies on the observation that an increase in the budget increases the consumption of the public good by the same amount for everybody, as was the case for $\delta = 1/N$. But the necessary increase in financing triggers redistributional conflicts that are essentially identical to those that are present when $\delta = 1/N$ since, except for δ, the model is the same.

This example suggests that, at least for some classes of utility functions, the factors that are conducive to progressivity when the budget is used to redistribute income are also conducive to progressivity when the budget is used solely to provide a public good.

VIII. Concluding Comments

Our intent in this chapter is to develop a positive theory of income taxation that generates tax schedules exhibiting marginal and average tax rates that rise with income. This feature is commonly found in many democratic countries, and in some states of the United States.

Economists and others have long speculated on the desirability of progressive taxes. Efforts to use theories of optimal taxation to explain the existence of progressivity have not been completely successful. Maximization of a Benthamite criterion very seldom leads to progressive tax structures. An alternative view, taken here, is that tax schedules are the outcome of a political equilibrium in which the majority imposes its will on the minority. All self-interested individuals would like to pay no taxes and to obtain positive redistribution from the government. Obviously this is not feasible for everybody. But in a democratic society in which tax schedules are determined by majority rule, the low- and middle-income majority can impose a certain level of redistribution on the more affluent minority. The analysis provides conditions under which this type of democracy leads to progressivity in the taxation of income.

A fundamental problem that arises once tax schedules are specified in a way that is sufficiently flexible to allow progressivity is that a majority-winning tax schedule need not exist. We derive conditions for the existance of a winning schedule within the set of quadratic schedules. An important condition for existence is that the ranking of gross incomes and of abilities be the same for all tax schedules. A sufficient condition for such identical ranking is the normality of both consumption and leisure. Under this condition, with some additional restrictions on the utility function and on the distribution of abilities, the individual with median ability is shown to be decisive. This result paves the way for finding conditions for progressivity, since it reduces this task to that of finding conditions under which the individual with median ability prefers progressivity. In contrast to the optimal taxation literature, we find that the set of circumstances under which the decisive voter, who maximizes utility, imposes progressivity is nonnegligible. Marginal progressivity is more likely (1) the larger the spread of the distribution of abilities in the population, (2) the smaller the labor supply response of the relatively more productive to an increase in marginal tax rates, and (3) the larger the difference between the ability of the decisive (median) voter and the mean ability of the community. (4) When the median works, larger disincentive effects of redistribution on labor force participation also increase progressivity. These conditions do not have to be satisfied separately; their combined effect is sufficient.

The methodology used to demonstrate the existence of a local majority-winning tax schedule can probably be used to extend this result to any tax schedule by viewing the quadratic schedule as a second-order Taylor expansion of a more general class of schedules. But we have not done that.

The tax schedules that result from our analysis reflect, mainly, skewness of the distribution of income that puts average income above median income, the variance of the distribution, and the effects of tax rates and redistribution on incentives to work. If these factors were identical across countries, and the franchise approximately the same, we would predict common tax schedules in democratic countries. Differences in tax schedules principally reflect possible differences in the voting rule that determines the franchise, differences in the distribution of income and ability, and differences in the effect of incentives, as measured in our analysis by the marginal effect of tax rates on labor supply.

Although the focus of the paper is on deriving conditions for progressivity when the budget is used to redistribute income, it is likely that similar conditions are conducive to progressivity when the budget is used to provide a public good. As illustrated in Section VII, even the provision of a pure public good is not independent of redistributional considerations, because of the need to decide on ways to finance it.

Some limitations of the analysis should be noted. Our model is static; income must be interpreted as lifetime income.[17] The budget is always balanced. We neglect migration and other open economy considerations that can limit progressivity over the time frame to which our model is most applicable. There is no capital, and therefore there are no taxes on capital.

Despite these limitations, the political economy model appears to be useful for understanding the determination of tax schedules and for showing that majority rule implies marginal progressivity to finance income redistribution and other expenditures under a relatively wide set of circumstances. The median or decisive voter chooses rising marginal tax rates if he can thereby reduce his own taxes without lowering the transfers he receives, or if he can increase the transfers he receives without increasing his own tax rates. Casual observations for many economies suggest that the voting process produces an outcome of this kind.

Appendix

1. *Derivation of Equations (15)*

Totally differentiating (13) with respect to τ, holding α constant, we obtain

$$\int_{x_0}^{x_u} \left[y + \tau x\left(\frac{\partial n}{\partial \tau} + r_\tau \frac{\partial n}{\partial r}\right) + 2\alpha x^2 n\left(\frac{\partial n}{\partial \tau} + r_\tau \frac{\partial n}{\partial r}\right)\right] dF$$
$$- r_\tau - \{\tau x_0 n(x_0) + \alpha[x_0 n(x_0)]^2\}\left(\frac{\partial x_0}{\partial \tau} + r_0 \frac{\partial x_0}{\partial r}\right) = 0$$

17. A recent application of the majority-rule paradigm to intertemporal redistribution and the determination of the public debt and deficits appears in Cukierman and Meltzer (1989), represented as Chapter 6 of this volume.

where the dependence of the various terms on x is not made explicit in the notation. Since $n(x_0) = 0$, the last term drops out. Equation (15a) follows by rearranging and by using equation (4) in the text.

Totally differentiating (13) with respect to α, holding τ constant, we obtain

$$\int_{x_0}^{x_u} \left[y^2 + \tau x\left(\frac{\partial n}{\partial \alpha} + r_\alpha \frac{\partial n}{\partial r}\right) + 2\alpha x^2 n\left(\frac{\partial n}{\partial \alpha} + r_\alpha \frac{\partial n}{\partial r}\right)\right] dF$$

$$- r_\alpha - \{\tau x_0 n(x_0) + \alpha[x_0 n(x_0)]^2\}\left(\frac{\partial x_0}{\partial \tau} + r_\tau \frac{\partial x_0}{\partial r}\right) = 0$$

Equation (15b) follows by noting that $n(x_0) = 0$, using (4), and by rearranging.

2. *Proof of Theorem 2 on the Existence and Characterization of a Global Median.*

It is convenient to break the proof into several lemmas.

Lemma A1: If condition (i) of theorem 2 is satisfied, s_m is a global majority winner against any change in tax schedule such that α and τ change in the same direction or such that only one of either α or τ changes.

Proof: Since $s_m \in S_m$, the median either dislikes or is indifferent to the change. Hence by lemma 5

$$P(y_m) \equiv -\Delta\alpha y_m^2 - \Delta\tau y_m + \Delta r < 0 \qquad \text{(A1)}$$

where

$$y_m \equiv y(s_m, x_m)$$

Multiplying (A1) by $-1/\Delta\alpha$

$$Q(y_m) \equiv y_m^2 + \frac{\Delta\tau}{\Delta\alpha} y_m - \frac{\Delta r}{\Delta\alpha} \qquad \text{(A2)}$$

(A1) and (A2) imply

$$Q(y_m) < 0 \quad \text{if } \Delta\alpha < 0 \qquad \text{(A3a)}$$

$$Q(y_m) > 0 \quad \text{if } \Delta\alpha > 0. \qquad \text{(A3b)}$$

Note that

$$Q'(y) = 2y + \frac{\Delta\tau}{\Delta\alpha} \qquad \text{(A4)}$$

which is positive for all $y \geq 0$ when $\Delta\tau$ and $\Delta\alpha$ have the same signs. Hence

$$Q(y) < Q(y_m) \quad \text{for all } \Delta\alpha < 0 \text{ and all } y < y_m \qquad \text{(A5a)}$$

$$Q(y) > Q(y_m) \quad \text{for all } \Delta\alpha > 0 \text{ and all } y > y_m \qquad \text{(A5b)}$$

(A3) and (A5) imply

$$P(y) < P(y_m) \quad \text{for all } \Delta\alpha < 0 \text{ and all } y < y_m \qquad \text{(A6a)}$$

$$P(y) < P(y_m) \quad \text{for all } \Delta\alpha > 0 \text{ and all } y > y_m \qquad \text{(A6b)}$$

But $P(y)$ is the change in the net income of an individual with prechange income y when he is not allowed to adjust his labor income. Condition (i) of theorem 2 therefore implies that

$$\text{when } \Delta\alpha > 0,\ s_m P_i s \quad \text{by all } y_i < y_m \qquad \text{(A7a)}$$

$$\text{when } \Delta\alpha < 0,\ s_m P_i s \quad \text{by all } y_i > y_m \qquad \text{(A7b)}$$

The notation $s_m P_i s$ should be read, "s_m is prefferred to s by an individual with prechange income y_i." Since the ranking of gross incomes is the same as that of abilities, (A7) implies that there is no change in schedule, such that $\Delta\alpha$ and $\Delta\tau$ have the same sign, that is strictly preferred by a majority.

When $\Delta\tau = 0$ and $\Delta\alpha \neq 0$, the fact that $s_m \in S_m$ in conjunction with lemma 5 implies

$$P(y_m) = -\Delta\alpha y_m^2 + \Delta r < 0$$

This implies

$$P(y) < P(y_m) \quad \text{for all } \Delta\alpha < 0 \text{ and all } y < y_m \qquad \text{(A8a)}$$

$$P(y) < P(y_m) \quad \text{for all } \Delta\alpha > 0 \text{ and all } y > y_m \qquad \text{(A8b)}$$

When $\Delta\tau \neq 0$ and $\Delta\alpha = 0$, the fact that $s_m \in S_m$ in conjunction with lemma 5 implies

$$P(y_m) = -\Delta\tau y_m + \Delta r < 0$$

This implies

$$P(y) < P(y_m) \quad \text{for all } \Delta\tau < 0 \text{ and all } y < y_m \qquad \text{(A9a)}$$

$$P(y) < P(y_m) \quad \text{for all } \Delta\tau > 0 \text{ and all } y > y_m \qquad \text{(A9b)}$$

(A8), (A9), and condition (i) of theorem 2 imply that there is no majority that strictly prefers changes of the type $\Delta\alpha = 0$ and $\Delta\tau \neq 0$ or $\Delta\alpha \neq 0$ and $\Delta\tau = 0$.

□

Lemma A2: If condition (i) of theorem 2 is satisfied, s_m is a global majority winner against any change in schedule of the type

$$\Delta\alpha > 0,\ \Delta\tau < 0$$

Proof: The fact that $s_m \in S_m$ and lemma 5 imply that (A1) holds. Since $\Delta\alpha > 0$, this is equivalent in turn to

$$Q(y_m) > 0 \tag{A10}$$

Since $\Delta\tau/\Delta\alpha < 0$, equation (A4) implies

$$Q'(y) = \begin{cases} >0 & y > \frac{1}{2}\left|\frac{\Delta\tau}{\Delta\alpha}\right| \\ =0 & y = \frac{1}{2}\left|\frac{\Delta\tau}{\Delta\alpha}\right| \\ <0 & y < \frac{1}{2}\left|\frac{\Delta\tau}{\Delta\alpha}\right| \end{cases}. \tag{A11}$$

(A10) and (A11) imply

$$Q(y) > Q(y_m) \quad \text{for all } y > y_m \text{ if } y_m > \frac{1}{2}\left|\frac{\Delta\tau}{\Delta\alpha}\right| \tag{A12a}$$

$$Q(y) > Q(y_m) \quad \text{for all } y < y_m \text{ if } y_m < \frac{1}{2}\left|\frac{\Delta\tau}{\Delta\alpha}\right| \tag{A12b}$$

$$Q(y) > Q(y_m) \quad \text{for all } y \neq y_m \text{ if } y_m = \frac{1}{2}\left|\frac{\Delta\tau}{\Delta\alpha}\right| \tag{A12c}$$

Since $Q(y)$ and $P(y)$ are inversely related for $\Delta\alpha > 0$, this implies that in all three cases at least 50 percent of the voters suffer (at prechange labor inputs) a decrease in net income that is larger than $P(y_m)$. Condition (i) of theorem 2 implies therefore that at least the same number of voters dislike changes of the type $\Delta\alpha > 0$, $\Delta\tau < 0$. Since the median either dislikes or is indifferent to the change, there is no change of the type $\Delta\alpha > 0$, $\Delta\tau < 0$ that is preferred by a majority to s_m. Hence s_m is a global majority winner against changes of this type. □

Lemma A3: s_m is a global majority winner against any change in schedule of the type

$$\Delta r \leqslant 0, \qquad \Delta\tau > 0, \qquad \Delta\alpha < 0$$

if conditions (i) and (ii) of theorem 2 are satisfied.

Proof: Due to condition (i) of theorem 2, all individuals with gross incomes in the open segment defined by (40) weakly dislike the change. Condition (ii) of theorem 2 ensures that for any b such that $\Delta r \leqslant 0$, $\Delta\tau > 0$, $\Delta\alpha < 0$ there is a majority that weakly dislikes the change. □

Lemma A4: s_m is a global majority winner against any change in schedule of the type

$$\Delta r > 0, \qquad \Delta\tau > 0, \qquad \Delta\alpha < 0$$

if conditions (i) and (iii) of theorem 2 are satisfied.

Proof: Since $\Delta r > 0$ and since $P(y_m) < 0$, so that $y_m < y^g_{c2}$, an argument similar to that which led to equation (29) in the text implies that

$$\frac{\bar{r}_\alpha}{\bar{r}_\tau} < b < \frac{\bar{r}_\alpha - y_m^2}{\bar{r}_\tau - y_m} \tag{A13}$$

provided the median is employed. Since s_m does not occur on the boundary of the TPF, $y_m = r_\alpha/r_\tau$ (equation (32)). Hence the condition in (A13) reduces to the restriction on b in condition (iii) of theorem 2. Since $\Delta r > 0$, $y^g_{c1} > 0$. Condition (i) of theorem 2 implies that all individuals with prechange gross incomes in the range

$$y^g_{c1}(b) + \varepsilon_1(b) < y < y^g_{c2}(b) - \varepsilon_2(b) \tag{A14}$$

dislike the change. Condition (iii) of theorem 2 implies that for all b's in the open interval defined by (A14) there is no majority in favor of the change. Hence s_m is a global majority winner against all changes of the type $\Delta r > 0$, $\Delta\tau > 0$, $\Delta\alpha < 0$, when the median works.

If the median does not work, the fact that he does not prefer the change implies that redistribution, r, goes down or does not change as a consequence of the change in tax schedule. Hence the case $\Delta r > 0$, $\Delta\tau > 0$, $\Delta\alpha < 0$ is not possible when the median does not work, so there is no need to consider it. □

The proof of theorem 2 is completed by combining lemmas A1 through A4.

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6

A Political Theory of Government Debt and Deficits in a Neo-Ricardian Framework

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Government expenditure is used for two main purposes—the provision of public goods and the redistribution of income. We focus here on the implications of redistribution for the size of the public debt, budgetary deficits, and surpluses. Since the focus is on redistribution, we abstract from the function of government as a provider of public goods and from issues that relate to minimization of the deadweight loss of taxation over time. This chapter can be viewed as complementary to the work of Robert Barro (1979), who proposed and tested a theory of public debt based on society's attempt to minimize the excess burden of taxation over time.

The main function of public debt is to redistribute the burden of taxation over time and across generations. In a neo-Ricardian world, such activity seems an idle exercise. Barro showed that, in the presence of an operative bequest motive and a perfect capital market, individuals totally undo the effects of debt-induced redistribution on consumption and welfare by adjusting their bequests appropriately. The existence of government debt in countries with developed capital markets, and the frequently stated belief that debt is a burden, is puzzling. Why do rational individuals complain about a burden that, according to Barro, does not occur?

The puzzle vanishes when individuals differ in abilities and therefore in wage earnings, and perhaps also in their initial nonhuman wealth. The reason is that some do not desire to leave positive bequests, and some would choose to borrow resources from future generations. As Allan Drazen (1978, footnote 1) has noted, individuals cannot obligate the future labor income of their descendants within

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