The Choice of Exchange-Rate Regime and Speculative Attacks*

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Abstract

We develop a framework that makes it possible to study, for the first time, the strategic interaction between the ex ante choice of exchange-rate regime and the likelihood of ex post currency attacks. The optimal regime is determined by a policymaker who trades off the loss from nominal exchange-rate uncertainty against the cost of adopting a given regime. This cost increases, in turn, with the fraction of speculators who attack the local currency. Searching for the optimal regime within the class of exchange-rate bands, we show that the optimal regime can be either a peg (a zero-width band), a free float (an infinite-width band), or a nondegenerate band of finite width. We study the effect of several factors on the optimal regime and on the probability of currency attacks. In particular, we show that a Tobin tax induces policymakers to set less flexible regimes. In our model, this generates an increase in the probability of currency attacks. **JEL Classification**: F31, D84

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1 Introduction

The literature on speculative attacks and currency crises can be broadly classified into first-generation models (Krugman 1979; Flood and Garber 1984) and second-generation models (Obstfeld 1994, 1996; Velasco 1997; Morris and Shin 1998). Recent surveys by Flood and Marion (1999) and Jeanne (2000) suggest that the main difference between the two generations of models is that, in first-generation models, the policies that ultimately lead to the collapse of fixed exchange-rate regimes are specified exogenously, whereas in second-generation models, policymakers play an active role in deciding whether or not to defend the currency against a speculative attack. In other words, second-generation models endogenize the policymakers' response to a speculative attack. As Jeanne (2000) points out, this evolution of the literature is similar to "the general evolution of thought in macroeconomics, in which government policy also evolved from being included as an exogenous variable in macroeconomic models to being explicitly modeled."

Although second-generation models explicitly model the policymakers' (ex post) response to speculative attacks, the initial (ex ante) choice of the exchange-rate regime (typically a peg) is treated in this literature as exogenous. As a result, the interdependence between ex post currency attacks and the ex ante choice of exchange-rate regime is ignored in this literature. A different strand of literature that focuses on optimal exchange-rate regimes (Helpman and Razin 1982; Devereux and Engel 1999) also ignores this effect by abstracting from the possibility of speculative attacks.[1]

This paper takes a first step toward bridging this gap by developing a model in which both the *ex ante choice of exchange-rate regime* and the *probability of ex post currency attacks* are determined endogenously. The model has three stages. In the first stage, prior to the realization of a stochastic shock to the freely floating exchange rate (the "fundamental" in the model), the policymaker chooses the exchange-rate regime. In the second stage, after the realization of fun-

[1] A related paper by Guembel and Sussman (2004) studies the choice of exchange-rate regime in the presence of speculative trading. Their model, however, does not deal with currency crises, as it assumes that policymakers are always fully committed to the exchange-rate regime. Also related is a paper by Jeanne and Rose (2002), which analyzes the effect of the exchange-rate regime on noise trading. However, they do not analyze the interaction between speculative trading and the abandonment of pre-announced exchange-rate regimes.

damentals, speculators decide whether or not to attack the exchange-rate regime. Finally, in the third stage, the policymaker decides whether to defend the regime or abandon it. Thus, relative to second-generation models, our model explicitly examines the ex ante choice of the exchange-rate regime. This makes it possible to rigorously examine, for the first time, the strategic interaction between the ex ante choice of regime and the probability of ex post currency attacks.

In order to model speculative attacks, we use the framework developed by Morris and Shin (1998) where each speculator observes a slightly noisy signal about the fundamentals of the economy, so that the fundamentals are not common knowledge among speculators. Besides making a step towards realism, this framework also has the advantage of eliminating multiple equilibria of the type that arise in second-generation models with common knowledge. In our context, this implies that the fundamentals of the economy uniquely determine whether a currency attack will or will not occur. This uniqueness result is important, since it establishes an unambiguous relation between the choice of exchange-rate regime and the likelihood of currency attacks.[2]

In general, characterizing the best exchange-rate regime is an extremely hard problem because the best regime may have an infinite number of arbitrary features. The difficulty is compounded by the fact that the exchange-rate regime affects, in turn, the strategic behavior of speculators vis-a-vis the policymaker and vis-a-vis each other. We therefore limit the search for the "best" regime to the class of explicit exchange-rate bands. This class of regimes is characterized by two parameters: the upper and the lower bounds of the band. The policymaker allows the exchange rate to move freely within these bounds but commits to intervene in the market and prevent the exchange rate from moving outside the band. Although the class of bands does not exhaust all possible varieties of exchange-rate regimes, it is nonetheless rather broad and includes

[2] The uniqueness result was first established by Carlsson and van Damme (1993), who use the term 'global games' to refer to games in which each player observes a different signal about the state of nature. Recently, the global games framework has been applied to study other issues that are related to currency crises, such as the effects of transparency (Heinemann and Illing 2002) and interest-rate policy (Angeletos, Hellwig and Pavan 2002). A similar framework has also been applied in other contexts (see, for example, Goldstein and Pauzner (2004) for an application to bank runs). For an excellent survey that addresses both applications and theoretical extensions (such as inclusion of public signals in the global games framework), see Morris and Shin (2003).

as special cases the two most commonly analyzed regimes: pegs (zero-width bands) and free floats (infinitely wide bands).[3] Our approach makes it possible to conveniently characterize the best regime in the presence of potential currency attacks within a substantially larger class of regimes than usually considered.

In order to focus on the main novelty of the paper, which is the strategic interaction between the ex ante choice of exchange-rate regime and the probability of ex post speculative attacks, we model some of the underlying macroeconomic structure in a reduced form.[4] A basic premise of our framework is that exporters and importers — as well as borrowers and lenders in foreign currency–denominated financial assets — dislike uncertainty about the level of the nominal exchange rate and that policymakers internalize at least part of this aversion. This premise is consistent with recent empirical findings by Calvo and Reinhart (2002). In order to reduce uncertainty and thereby promote economic activity, the policymaker may commit to an exchange-rate band or even to a peg. Such commitment, however, is costly because maintenance of the currency within the band occasionally requires the policymaker to use up foreign exchange reserves or deviate from the interest-rate level that is consistent with other domestic objectives. The cost of either option rises if the exchange rate comes under speculative attack. If the policymaker decides to exit the band and avoid the costs of defending it, he loses credibility. The optimal exchange-rate regime reflects, therefore, a trade-off between reduction of exchange-rate uncertainty and the cost of committing to an exchange-rate band or a peg. This trade-off is in the spirit of the "escape clause" literature (Lohmann 1992; Obstfeld 1997).

By explicitly recognizing the interdependence between speculative attacks and the choice of exchange-rate regime, our framework yields a number of novel predictions about the optimal exchange-rate regime and about the likelihood of a currency attack. For instance, we analyze the

[3] Garber and Svensson (1995) note that "fixed exchange-rate regimes in the real world typically have explicit finite bands within which exchange rates are allowed to fluctuate." Such intermediate regimes have been adopted during the 1990s by a good number of countries, including Brazil, Chile, Colombia, Ecuador, Finland, Hungary, Israel, Mexico, Norway, Poland, Russia, Sweden, The Czech Republic, The Slovak Republic, Venezuela, and several emerging Asian countries.

[4] For the same reason, we also analyze a three-stage model instead of a full-fledged dynamic framework. In utilizing this simplification we follow Obstfeld (1996) and Morris and Shin (1998), who analyze reduced-form two-stage models.

effect of a Tobin tax on short-term intercurrency transactions that was proposed by Tobin (1978) as a way of reducing the profitability of speculation against the currency and thereby lowering the probability of currency crises. We show that such a tax induces policymakers to set narrower bands in order to achieve more ambitious reductions in exchange-rate uncertainty.[5] When this endogeneity of the regime is considered, the tax, in our model, actually raises the probability of currency attacks. Thus, though it is still true that the tax lowers the likelihood of currency crises for a given band, the fact that it induces less flexible bands attracts more speculative attacks. The paper also shows that, in spite of the increase in the likelihood of a crisis, the imposition of a Tobin tax improves the objectives of policymakers. Using the same structure, the paper analyzes the effects of other factors — such as the aversion to exchange-rate uncertainty, the variability in fundamentals and the tightness of commitment — on the choice of exchange-rate regime and on the probability of currency attacks.

As a by-product, the paper also contributes to the literature on target zones and exchange-rate bands. The paper focuses on the trade-offs that determine the optimal band width by analyzing the strategic interaction between the ex ante choice of exchange-rate regime and the behavior of speculators. To this end, it abstracts from the effect of a band on the behavior of the exchange rate within the band, which is a main focus of the traditional target zone literature.[6] We are aware of only three other papers that analyze the optimal width of the band: Sutherland (1995), Miller and Zhang (1996) and Cukierman, Spiegel and Leiderman (2004). The first two papers do not consider the possibility of realignments or the interaction between currency attacks and the optimal width of the band. The third paper incorporates the possibility of realignments, but abstracts from the issue of speculative attacks.

The remainder of this paper is organized as follows. Section 2 presents the basic framework.

[5] This result is also consistent with the flexibilization of exchange-rate regimes following the gradual elimination of restrictions on capital flows in the aftermath of the Bretton Woods system.

[6] This literature orignated with a seminal paper by Krugman (1991) and continued with many other contributions, such as Bertola and Caballero (1992) and Bertola and Svensson (1993). See Garber and Svensson (1995) for an extensive literature survey. Because of the different focus, our paper and the target zone literature from the early 1990s complement each other.

Section 3 is devoted to deriving the equilibrium behavior of speculators and of the policymaker and to characterizing the equilibrium properties of the exchange-rate regime. Section 4 provides comparative statics analysis and discusses its implications for various empirical issues, including the effects of a Tobin tax. Section 5 concludes. All proofs are in the Appendix.

2 The Model

Consider an open economy in which the initial level of the nominal exchange rate (defined as the number of units of domestic currency per one unit of foreign currency) is e_{-1}. Absent policy interventions and speculation, the new level of the unhindered nominal exchange rate e reflects various shocks to the current account and the capital account of the balance of payments. The excluded behavior of speculators and government interventions is the focus of the model in this paper. For the purpose of this paper, it turns out that it is more convenient to work with the laissez-faire rate of change in e, $x \equiv (e - e_{-1})/e_{-1}$, rather than with its level. We assume that x is drawn from a distribution function $f(x)$ on $\Re$ with c.d.f. $F(x)$. We make the following assumption on $f(x)$:

Assumption 1: The function $f(x)$ is unimodal with a mode at $x = 0$. That is, $f(x)$ is increasing for all $x < 0$ and decreasing for all $x > 0$.

Assumption 1 states that large rates of change in the freely floating exchange rate (i.e., large depreciations when $x > 0$ and large appreciations when $x < 0$) are less likely than small rates of change. This is a realistic assumption and, as we shall see later, it is responsible for some main results in the paper.

2.1 The Exchange-Rate Band

A basic premise of this paper is that policymakers dislike nominal exchange-rate uncertainty. This is because exporters, importers, as well as lenders and borrowers in foreign currency face higher exchange-rate risks when there is more uncertainty about the nominal exchange rate. By raising the foreign exchange risk premium, an increase in exchange-rate uncertainty reduces international flows of goods and financial capital. Policymakers, who wish to promote economic activity, internalize at least part of this aversion to uncertainty and thus have an incentive to limit it.[7]

In general, there are various conceivable institutional arrangements for limiting exchange-rate uncertainty. In this paper we search for an optimal institutional arrangement within the class of bands. This class is quite broad and includes pegs (bands of zero width) and free floats (bands of infinite width) as special cases. Under this class of arrangements, the policymaker sets an exchange-rate band $[\underline{e}, \overline{e}]$ around the pre-existing nominal exchange rate, e_{-1}. The nominal exchange rate is then allowed to move freely within the band in accordance with the realization of the laissez-faire exchange rate, e. But if this realization is outside the band, the policymaker is committed to intervene and keep the exchange rate at one of the boundaries of the band.[8] Thus, given e_{-1}, the exchange-rate band induces a permissible range of rates of change in the exchange rate, $[\underline{\pi}, \overline{\pi}]$, where $\underline{\pi} \equiv \frac{\underline{e}-e_{-1}}{e_{-1}} < 0$ and $\overline{\pi} \equiv \frac{\overline{e}-e_{-1}}{e_{-1}} > 0$. Within this range, the domestic currency is allowed to appreciate if $x \in [\underline{\pi}, 0)$ and to depreciate if $x \in [0, \overline{\pi})$. In other words, $\underline{\pi}$ is the maximal rate of appreciation and $\overline{\pi}$ is the maximal rate of depreciation that the exchange-rate band allows.[9]

But leaning against the trends of free exchange-rate markets is costly. To defend a currency under attack, policymakers have to deplete their foreign exchange reserves (Krugman 1979) or put up with substantially higher domestic interest rates (Obstfeld 1996). The resulting cost is $C(y, \alpha)$, where y is the absolute size of the disequilibrium that the policymaker tries to maintain (i.e., $x - \overline{\pi}$ if $x > \overline{\pi}$ or $\underline{\pi} - x$ if $x < \underline{\pi}$) and α is the fraction of speculators who attack the band (we normalize the mass of speculators to 1). Following Obstfeld (1996) and Morris and Shin (1998), we assume that $C(y, \alpha)$ is increasing in both y and α. Also, without loss of generality, we assume that $C(0, 0) = 0$.

Admittedly, this cost function is reduced form in nature. Nonetheless, it captures the important aspects of reality that characterize defense of the exchange rate. In reality, the cost of defending the exchange rate stems from loss of reserves following intervention in the exchange-rate

[7] Admittedly, some of those risks may be insured by means of future currency markets. However, except perhaps for some of the major key currencies, such markets are largely nonexistent, and when they do exist the insurance premia are likely to be prohibitively high.

[8] This intervention can be operationalized by buying or selling foreign currency in the market, by changing the domestic interest rate, or by doing some of both.

[9] Note that, when $\underline{\pi} = \overline{\pi} = 0$, the band reduces to a peg; when $\underline{\pi} = -\infty$ and $\overline{\pi} = \infty$, it becomes a free float.

market and from changes in the interest rate. The amount of reserves depleted in an effort to defend the currency is increasing in the fraction of speculators, α, who run on the currency. The increase in the interest rate needed to prevent depreciation is higher the higher are the disequilibrium, y, that the policymaker is trying to maintain, and the fraction of speculators, α, who attack the currency. Hence the specification of $C(y,\alpha)$ captures in a reduced-form manner the important effects that would be present in many reasonable and detailed specifications. In addition, because of its general functional form, $C(y,\alpha)$ can accommodate a variety of different structural models.

If policymakers decide to avoid the cost $C(y,\alpha)$ by exiting the band, they lose some credibility. This loss makes it harder to achieve other goals either in the same period or in the future (e.g., committing to a low rate of inflation or to low rates of taxation, accomplishing structural reforms, etc.). We denote the present value of this loss by δ. Hence δ characterizes the policymaker's aversion to realignments. Obviously, the policymaker will maintain the band only when $C(y,\alpha) \leq \delta$. Otherwise, the policymaker will exit the band and incur the cost of realignment, δ. The policymaker's cost of adopting an exchange-rate band for a given x is therefore $Min\{C(y,\alpha),\delta\}$.

We formalize the trade-off between uncertainty about the nominal exchange rate and the cost of adopting a band by postulating that the policymaker's objective is to select the bounds of the band, $\underline{\pi}$ and $\overline{\pi}$, to maximize

$$V(\underline{\pi},\overline{\pi}) = -AE\left|\pi - E\pi\right| - E\left[Min\{C(y,\alpha),\delta\}\right], \qquad A>0, \tag{1}$$

where π is the actual rate of change in the nominal exchange rate (under laissez-faire, $\pi = x$).

We think of the policymaker's maximization problem mostly as a positive description of how a rational policymaker might approach the problem of choosing the band width. The second component of V is simply the policymaker's expected cost of adopting an exchange-rate band. The first component of V represents the policymaker's aversion to nominal exchange-rate *uncertainty*, measured in terms of the expected absolute value of unanticipated nominal depreciations/appreciations.[10] The parameter A represents the relative importance that the policymaker

[10] It is important to note that the policymaker is averse to excahnge-rate *uncertainty* and not to actual exchange-rate *variability* (see Cukierman and Wachtel (1982) for a general distinction between uncertainty and variability).

assigns to reducing exchange-rate uncertainty and is likely to vary substantially across economies, depending on factors like the degree of openness of the economy, its size, the fraction of financial assets and liabilities owned by domestic producers and consumers that are denominated in foreign exchange, and the fraction of foreign trade that is invoiced in foreign currency (McKinnon 2000; Gylfason 2000; Wagner 2000). All else equal, residents of small open economies are more averse to nominal exchange-rate uncertainty than residents of large and relatively closed economies like the United States or the Euro area. Hence, a reasonable presumption is that A is larger in small open economies than in large, relatively closed economies.

2.2 Speculators

We model speculative behavior using the Morris and Shin (1998) apparatus. There is a continuum of speculators, each of whom can take a position of at most one unit of foreign currency. The total mass of speculators is normalized to 1. When the exchange rate is either at the upper bound of the band, $\overline{e}$, or at the lower bound, $\underline{e}$, each speculator i independently observes a noisy signal, θ_i, on the exchange rate that would prevail under laissez-faire. Specifically, we assume that

$$\theta_i = x + \varepsilon_i, \tag{2}$$

where ε_i is a white noise, that is independent across speculators and distributed uniformly on the interval $[-\varepsilon, \varepsilon]$. The conditional density of x given a signal θ_i is:

$$f(x \mid \theta_i) = \frac{f(x)}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \tag{3}$$

In what follows, we focus on the case where ε is small so that the signals that speculators observe are "almost perfect."

Based on θ_i, each speculator i decides whether or not to attack the currency. If the exchange rate is at $\underline{e}$, speculator i can shortsell the foreign currency at the current (high) price $\underline{e}$ and then buy

Indeed, this is the reason for commiting to a band ex ante: without commitment, there is a time inconsistency problem (Kydland and Prescott 1977; Barro and Gordon 1983), so the market will correctly anticipate that — since he is not averse to predictable variability — the policymaker will have no incentive to intervene ex post after the realization of x.

the foreign currency on the market to clear his position. Denoting by t the nominal transaction cost associated with switching between currencies, the speculator's net payoff is $\underline{e} - e - t$, if the policymaker fails to defend the band and the exchange rate falls below $\underline{e}$. Otherwise, the payoff is $-t$. Likewise, if the exchange rate is at $\overline{e}$, speculator i can buy the foreign currency at the current (low) price $\overline{e}$. Hence, the speculator's net payoff is $e - \overline{e} - t$ if the policymaker exits the band and the exchange rate jumps to $e > \overline{e}$. If the policymaker successfully defends the band, the payoff is $-t$. If the speculator does not attack the band, his payoff is 0.[11] To rule out uninteresting cases, we make the following assumption:

Assumption 2: $C\left(\frac{t}{e_{-1}}, 0\right) < \delta$.

This assumption ensures that speculators will always attack the band if they believe that the policymaker is not going to defend it.

2.3 The Sequence of Events and the Structure of Information

The sequence of events unfolds as follows:

- Stage 1: The policymaker announces a band around the existing nominal exchange rate and commits to intervene when $x < \underline{\pi}$ or $x > \overline{\pi}$.

- Stage 2: The "free float" random shock, x, is realized. There are now two possible cases:

  (i) If $\underline{\pi} \leq x \leq \overline{\pi}$, the nominal exchange rate is determined by its laissez-faire level: $e = (1 + x)e_{-1}$.

  (ii) If $x < \underline{\pi}$ or $x > \overline{\pi}$, then the exchange rate is at $\underline{e}$ or at $\overline{e}$, respectively. Simultaneously, each speculator i gets the signal θ_i on x and decides whether or not to attack the band.

- Stage 3: The policymaker observes x and the fraction of speculators who decide to attack the band, α, and then decides whether or not to defend the band. If he does, the exchange rate stays at the boundary of the band and the policymaker incurs the cost $C(y, \alpha)$. If

[11] In order to focus on speculation against the band, we abstract from speculative trading within the band. Thus, the well-known "honeymoon effect" (Krugman 1991) is absent from the model.

the policymaker exits the band, the exchange rate moves to its freely floating rate and the policymaker incurs a credibility loss of δ.[12]

3 The Equilibrium

To characterize the perfect Bayesian equilibrium of the model, we solve the model backwards. First, if $x < \underline{\pi}$ or $x > \overline{\pi}$ then, given α, the policymaker decides in Stage 3 whether or not to continue to maintain the band. Second, given the signals that they observe in Stage 2, speculators decide whether or not to attack the band. Finally, in Stage 1, prior to the realization of x, the policymaker sets the exchange-rate regime.

3.1 Speculative Attacks

When $x \in [\underline{\pi}, \overline{\pi}]$, the exchange rate is determined solely by its laissez-faire level. In contrast, when $x < \underline{\pi}$ or $x > \overline{\pi}$, the exchange rate moves to one of the boundaries of the band. Then, speculators may choose to attack the band if they expect that the policymaker will eventually exit the band. But since speculators do not observe x and α directly, each speculator needs to use his own signal in order to assess the policymaker's decision on whether to continue to defend the band or abandon it. Lemma 1 characterizes the equilibrium in the resulting game.

Lemma 1 *Suppose that speculators have almost perfect information, i.e., $\varepsilon \to 0$. Then,*

[12] The events at Stages 2 and 3 are similar to those in Morris and Shin (1998) and follow the implied sequence of events in Obstfeld (1996). The assumptions imply that speculators can profit from attacking the currency if there is a realignment, and that the policymaker realigns only if the fraction of speculators who attack is sufficiently large. These realistic features are captured in the model in a reduced-form manner. One possible way to justify these features within our framework is as follows: Initially (at Stage 2), the exchange rate policy is on "automatic pilot" (the result, say, of a short lag in decision making or in the arrival of information), so the policymaker intervenes automatically as soon as the exchange rate reaches the boundaries of the band. Speculators buy foreign currency or shortsell it at this point in the hope that a realignment will take place. In Stage 3, the policymaker re-evaluates his policy by comparing $C(y, \alpha)$ and δ. If $C(y, \alpha) > \delta$, he exits from the band and speculators make a profit on the difference between the price at Stage 2 and the new price set in Stage 3. For simplicity, we assume that the cost of intervention in Stage 2 is zero. In a previous version we also analyzed the case where the cost of intervention in Stage 2 is positive but found that all our results go through.

(i) *When the exchange rate reaches the upper (lower) bound of the band, there exists a unique perfect Bayesian equilibrium such that each speculator attacks the band if and only if the signal that he observes is above some threshold* $\overline{\theta}^*$ *(below some threshold* $\underline{\theta}^*$*).*

(ii) *The thresholds* $\overline{\theta}^*$ *and* $\underline{\theta}^*$ *are given by* $\overline{\theta}^* = \overline{\pi} + r$ *and* $\underline{\theta}^* = \underline{\pi} - r$*, where* r *is positive and is defined implicitly by*

$$C\left(r, 1 - \frac{t}{re_{-1}}\right) = \delta,$$

and r *is increasing in* t *and in* δ*.*

(iii) *In equilibrium, all speculators attack the upper (lower) bound of the band and the policymaker realigns it if and only if* $x > \overline{\theta}^* = \overline{\pi} + r$ $(x < \underline{\theta}^* = \underline{\pi} - r)$*. The probability of a speculative attack is*

$$P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r)).$$

The proof of Lemma 1 (along with proofs of all other results) is in the Appendix. The uniqueness result in part (i) follows from arguments similar to those in Carlsson and van Damme (1993) and Morris and Shin (1998) and is based on an iterative elimination of dominated strategies. The idea is as follows. Suppose that the exchange rate has reached $\overline{e}$ (the logic when the exchange rate reaches $\underline{e}$ is analogous). When θ_i is sufficiently large, speculator i correctly anticipates that x is such that the policymaker will surely exit the band even if no speculator attacks it. Hence, it is a dominant strategy for speculator i to attack.[13] But now, if θ_i is slightly lower, speculator i realizes that a large fraction of speculators must have observed even higher signals and will surely attack the band. From that, speculator i concludes that the policymaker will exit the band even at this slightly lower signal, so it is again optimal to attack it. This chain of reasoning proceeds further, where each time we lower the critical signal above which speculator i will attack $\overline{e}$. Likewise, when θ_i is sufficiently low, speculator i correctly anticipates that x is so low that the profit from attacking is below the transaction cost t even if the policymaker will surely exit the band. Hence, it is a dominant strategy not to attack at $\overline{e}$. But then, if θ_i is slightly higher, speculator i correctly

[13] The existence of a region in which speculators have dominant strategies is crucial for deriving a unique equilibrium (Chan and Chiu 2002).

infers that a large fraction of speculators must have observed even lower signals and will surely not attack at $\overline{e}$. From that, speculator i concludes that the policymaker will successfully defend $\overline{e}$ so again it is optimal not to attack. Once again, this chain of reasoning proceeds further, where each time we raise the critical signal below which the speculator will not attack $\overline{e}$.

As $\varepsilon \to 0$, the critical signal above which speculators attack $\overline{e}$ coincides with the critical signal below which they do not attack it. This yields a unique threshold signal $\overline{\theta}^*$ such that all speculators attack $\overline{e}$ if and only if they observe signals above $\overline{\theta}^*$. Similar arguments establish the existence of a unique threshold signal $\underline{\theta}^*$ such that all speculators attack $\underline{e}$ if and only if they observe signals below $\underline{\theta}^*$.

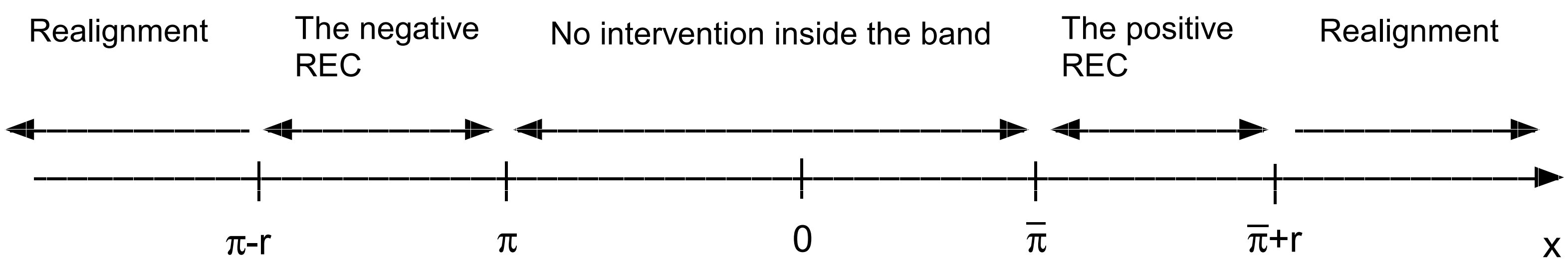
Having characterized the behavior of speculators, we turn next to the implications of this behavior for the exchange-rate band. Part (iii) of Lemma 1 implies that the exchange-rate band gives rise to two *Ranges of Effective Commitment* (RECs) such that the policymaker intervenes in the exchange-rate market and defends the band if and only if x falls inside one of these ranges. The positive REC is equal to $[\overline{\pi}, \overline{\pi} + r]$; when $x \in [\overline{\pi}, \overline{\pi} + r]$, the policymaker ensures that the rate of depreciation will not exceed $\overline{\pi}$. The negative REC is equal to $[\underline{\pi} - r, \underline{\pi}]$; when $x \in [\underline{\pi} - r, \underline{\pi}]$, the policymaker ensures that the rate of appreciation will not exceed the absolute value of $\underline{\pi}$. When $x < \underline{\pi} - r$ or when $x > \overline{\pi} + r$, the policymaker exits the band and — despite his earlier announcement — allows a realignment. Finally, when $x \in [\underline{\pi}, \overline{\pi}]$, the policymaker allows the exchange rate to move freely. These five ranges of x are illustrated in Figure 1.

Part (ii) of Lemma 1 indicates that r is independent of $\underline{\pi}$ and $\overline{\pi}$. This means that the actual size of the two RECs does not depend on how wide the band is. But, by choosing $\underline{\pi}$ and $\overline{\pi}$ appropriately, the policymaker can shift the two RECs either closer to or away from zero. Part (ii) of Lemma 1 also shows that r increases with t and with δ: a realignment is less likely when it is more costly for speculators to attack the band and also when a realignment is more costly for the policymaker.

The discussion is summarized in Proposition 1.[14]

[14]It can be shown that the equilibrium described in the proposition is also an equilibrium in a model where the policymaker receives a noisy signal of x (as does each of the speculators) rather than a precise observation of it. Moreover, when the signal observed by the policymaker is sufficiently precise relative to the signals observed by

Figure 1: Illustrating the exchange rate band



Proposition 1 *The exchange-rate band gives rise to a positive range of effective commitment (REC),* $[\overline{\pi}, \overline{\pi}+r]$*, and a negative REC,* $[\underline{\pi}-r, \underline{\pi}]$*, where* r *is defined in Lemma 1.*

- *When* x *falls inside the positive (negative) REC, the policymaker defends the currency and ensures that the maximal rate of depreciation (appreciation) is* $\overline{\pi}$ *(*$\underline{\pi}$*).*

- *When* x *falls below the negative REC, above the positive REC, or inside the band, the policymaker lets the exchange rate move freely in accordance with market forces.*

- *The width of the two RECs,* r*, increases with* t *and with* δ *but is independent of the boundaries of the band,* $\underline{\pi}$ *and* $\overline{\pi}$*.*

3.2 The Choice of Band Width

In order to characterize the equilibrium exchange-rate regime, we first need to write the policymaker's objective function, $V(\underline{\pi}, \overline{\pi})$, more explicitly. The first component in $V(\underline{\pi}, \overline{\pi})$ represents the policymaker's loss from exchange-rate uncertainty. This term depends on the expected rate of change in the exchange rate, $E\pi$, which in turn depends on the policymaker's choices, $\underline{\pi}$ and $\overline{\pi}$.

At first blush one may think that, since $\underline{\pi}$ is the maximal rate of appreciation and $\overline{\pi}$ is the maximal rate of depreciation, $E\pi$ will necessarily lie between $\underline{\pi}$ and $\overline{\pi}$. However, since the policymaker does not always defend the band, $E\pi$ may in principle fall outside the interval $[\underline{\pi}, \overline{\pi}]$. For example, if $\overline{\pi}$ is sufficiently small and if $f(x)$ has a larger mass in the positive range of x than in its negative range, then $E\pi$ will be high. If this asymmetry of $f(x)$ is sufficiently strong, $E\pi$ will actually be higher than $\overline{\pi}$. Consequently, in writing $V(\underline{\pi}, \overline{\pi})$ we need to distinguish between five possible cases depending on whether $E\pi$ falls inside the interval $[\underline{\pi}, \overline{\pi}]$, inside one of the two RECs, below the negative REC, or above the positive REC.

To simplify the exposition, from now on we will restrict attention to the following case:

Assumption 3: The distribution $f(x)$ *is symmetric around* 0*.*

Assumption 3 considerably simplifies the following analysis. It implies that the mean of x is 0 and hence that, on average, the freely floating exchange rate does not generate pressures for

speculators, this equilibrium will be the unique equilibrium, just as in our model.

either appreciations or depreciations. We can now prove the following lemma.[15]

Lemma 2 *Given Assumptions 1 and 3,* $E\pi \in [\underline{\pi}, \overline{\pi}]$.

Given Lemma 2, the measure of exchange-rate uncertainty is given by:

$$\begin{aligned} E\left|\pi - E\pi\right| &= -\int_{-\infty}^{\underline{\pi}-r} (x - E\pi)\, dF(x) - \int_{\underline{\pi}-r}^{\underline{\pi}} (\underline{\pi} - E\pi)\, dF(x) - \int_{\underline{\pi}}^{E\pi} (x - E\pi)\, dF(x) \\ &\quad + \int_{E\pi}^{\overline{\pi}} (x - E\pi)\, dF(x) + \int_{\overline{\pi}}^{\overline{\pi}+r} (\overline{\pi} - E\pi)\, dF(x) + \int_{\overline{\pi}+r}^{\infty} (x - E\pi)\, dF(x). \end{aligned} \tag{4}$$

Equation (4) implies that the existence of a band affects uncertainty only through its effect on the two RECs. Using (1) and (4), the expected payoff of the policymaker, given $\underline{\pi}$ and $\overline{\pi}$, becomes

$$\begin{aligned} V(\underline{\pi}, \overline{\pi}) &= A\left[\int_{-\infty}^{\underline{\pi}-r} (x - E\pi)\, dF(x) + \int_{\underline{\pi}-r}^{\underline{\pi}} (\underline{\pi} - E\pi)\, dF(x) + \int_{\underline{\pi}}^{E\pi} (x - E\pi)\, dF(x) \right. \\ &\quad \left. - \int_{E\pi}^{\overline{\pi}} (x - E\pi)\, dF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r} (\overline{\pi} - E\pi)\, dF(x) - \int_{\overline{\pi}+r}^{\infty} (x - E\pi)\, dF(x)\right] \\ &\quad - \int_{-\infty}^{\underline{\pi}-r} \delta dF(x) - \int_{\underline{\pi}-r}^{\underline{\pi}} c(\underline{\pi} - x) dF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r} c(x - \overline{\pi}) dF(x) - \int_{\overline{\pi}+r}^{\infty} \delta dF(x), \end{aligned} \tag{5}$$

where $c(\cdot) \equiv C(\cdot, 0)$. The last line in (5) represents the expected cost of adopting a band. As Lemma 1 shows, when x falls inside the two RECs, no speculator attacks the band; hence the policymaker's cost of intervention in the exchange-rate market is $c(\underline{\pi} - x)$ when $x \in [\underline{\pi} - r, \underline{\pi}]$ or $c(x - \overline{\pi})$ when $x \in [\overline{\pi}, \overline{\pi} + r]$. When either $x < \underline{\pi} - r$ or $x > \overline{\pi} + r$, there are realignments and so the policymaker incurs a credibility loss δ.

The policymaker chooses the boundaries of the band, $\underline{\pi}$ and $\overline{\pi}$, so as to maximize $V(\underline{\pi}, \overline{\pi})$. The next lemma enables us to simplify the characterization of the optimal band.

Lemma 3 *Given Assumption 3, the equilibrium exchange-rate band will be symmetric around* 0 *in the sense that* $-\underline{\pi} = \overline{\pi}$. *Consequently,* $E\pi = 0$.

Since the band is symmetric, it is sufficient to characterize the optimal value of the upper bound of the band, $\overline{\pi}$. By symmetry, the lower bound will then be equal to $-\overline{\pi}$. Given that

[15] It should be noted that the general qualitative spirit of our analysis extends to the case where $f(x)$ is asymmetric. But the various mathematical expressions and conditions become more complex.

$c(0) \equiv C(0,0) = 0$, it follows that $c(r) = \int_{\overline{\pi}}^{\overline{\pi}+r} c'(x-\overline{\pi})dx$. Together with the fact that at the optimum, $E\pi = 0$, the derivative of $V(\underline{\pi}, \overline{\pi})$ with respect to $\overline{\pi}$ is:

$$\frac{\partial V(\underline{\pi}, \overline{\pi})}{\partial \overline{\pi}} = -A \int_{\overline{\pi}}^{\overline{\pi}+r} \left(f(x) - f(\overline{\pi}+r)\right) dx + \int_{\overline{\pi}}^{\overline{\pi}+r} c'(x-\overline{\pi}) \left(f(x) - f(\overline{\pi}+r)\right) dx + \delta f(\overline{\pi}+r). \tag{6}$$

Equation (6) shows that, by altering $\overline{\pi}$, the policymaker trades off the benefits of reducing exchange-rate uncertainty against the cost of maintaining a band. The term in the first line of (6) is the marginal effect of $\overline{\pi}$ on exchange-rate uncertainty. Since by Assumption 1, $f(x) - f(\overline{\pi}+r) > 0$ for all $x \in [\overline{\pi}, \overline{\pi}+r]$, this term is negative and represents the marginal cost of raising $\overline{\pi}$. This marginal cost arises because, when $\overline{\pi}$ is raised, the positive REC over which the exchange rate is kept constant shifts farther away from the center rate to a range of shocks that is less likely (by Assumption 1). Hence, the band becomes less effective in reducing exchange-rate uncertainty. The second line in (6) represents the marginal effect of raising $\overline{\pi}$ on the expected cost of adopting a band. By Assumption 1 and since $c'(\cdot) > 0$, the integral term is positive, implying that raising $\overline{\pi}$ makes it less costly to defend the band. This is because it is now less likely that the policymaker will actually have to defend the band. The term involving δ is also positive since increasing $\overline{\pi}$ slightly lowers the likelihood that the exchange rate will move outside the positive REC and lead to a realignment.

Proposition 2 provides sufficient conditions for alternative types of exchange-rate regimes:

Proposition 2 *The equilibrium exchange-rate band has the following properties:*

(i) A free float: *If $A \le c'(y)$ for all y, then $\underline{\pi} = -\infty$ and $\overline{\pi} = \infty$, so the optimal regime is a free float.*

(ii) A nondegenerate band: *If*

$$\underline{A}(r) \equiv \frac{\delta(1-F(r)) + \int_0^r c(x)dF(x)}{\int_0^r x dF(x)} < A < \frac{\delta f(r) - \int_0^r c(x) f'(x) dx}{\int_0^r \left(f(x) - f(r)\right) dx} \equiv \overline{A}(r), \tag{7}$$

then $-\infty < \underline{\pi} < 0 < \bar{\pi} < \infty$. Hence, the optimal regime is a nondegenerate band.

(iii) A peg: *If $V(\underline{\pi}, \overline{\pi})$ is concave and $A > \overline{A}(r)$, then $\underline{\pi} = \bar{\pi} = 0$, and so the optimal regime is a peg.*

Part (i) of Proposition 2 states that when the policymaker has sufficiently little concern for nominal exchange-rate uncertainty (i.e., A is small relative to $c'(y)$), then he sets a free float and completely avoids the cost of maintaining a band. Part (ii) of the proposition identifies an intermediate range of values of A for which the optimal regime is a nondegenerate band. When A is below the upper bound of this range, $\overline{A}(r)$, it is optimal to increase $\overline{\pi}$ above zero and thus the optimal regime is not a peg. When A is above the lower bound of this range, $\underline{A}(r)$, a peg is better than a free float. Thus, when A is inside this range, the optimal regime is a nondegenerate band.[16] Part (iii) of Proposition 2 states that if the policymaker is highly concerned with nominal exchange-rate uncertainty (i.e., $A > \overline{A}(r)$), then his best strategy is to adopt a peg.[17]

4 Comparative Statics and Empirical Implications

In this section, we examine the comparative statics properties of the optimal band under the assumption that there is an internal solution (i.e., the optimal regime is a non-degenerate band). This means that the solution is obtained by equating the expression in (6) to zero. To assure that such a solution exists, we assume that $A > c'(y)$ for all y. In the appendix, we derive conditions for a unique internal solution.

4.1 The Effects of Restrictions on Capital Flows and of a Tobin Tax

During the last three decades there has been a worldwide gradual lifting of restrictions on currency flows and on related capital account transactions. One consequence of this trend is a reduction in the transaction cost of foreign exchange transactions (t in terms of the model), making it easier

[16] Note that the range specified in equation (7) represents only a (restrictive) sufficient condition for the optimal regime to be a nondegenerate band. Thus, the actual range in which the regime is a nondegenerate band should be larger. Also, note that the range in (7) is usually nonempty. For example, when $C(y, \alpha) = y + \alpha$ and $f(x)$ is a triangular symmetric distribution function with supports $-\overline{x}$ and $\overline{x}$ ($\overline{x} > 0$), this range is nonempty for all $r < \overline{x}$. For brevity, we do not demonstrate this explicitly in the paper.

[17] Note that a peg does not mean that the exchange rate is fixed under all circumstances. When the absolute value of x exceeds r, the policymaker abandons the peg and the exchange rate is realigned. Hence, under a peg, the exchange rate is fixed for all $x \in [-r, r]$. Given Assumption 1, such "small" shocks are more likely than big ones, so when A is large it is optimal for the policymaker to eliminate these shocks by adopting a peg.

for speculators to move funds across different currencies and thereby facilitating speculative attacks. To counteract this tendency, some economists proposed to "throw sand" into the wheels of unrestricted international capital flows. In particular, Tobin (1978) proposed a universal tax on short-term intercurrency transactions in order to reduce the profitability of speculation against the currency and hence the probability of crises. This idea was met with skepticism owing mainly to difficulties of implementation. Yet, by and large the consensus is that, subject to feasibility, the tax can reduce the probability of attack on the currency. Recent evaluations appear in Eichengreen, Tobin and Wyplosz (1995), Jeanne (1996), Haq, Kaul and Grunberg (1996), Eichengreen (1999), and Berglund et al. (2001).

The main objective of this section is to examine the consequences of such a tax and of the lifting of restrictions on capital flows when the choice of exchange-rate regime is endogenous.

Proposition 3 *Suppose that, following a lifting of restrictions on currency flows and capital account transactions, the transaction cost of switching between currencies, t, decreases. Then:*

(i) When the policymaker's problem has a unique interior solution, $\bar{\pi}$ and $\underline{\pi}$ shift away from zero and so the band becomes wider. Moreover, the probability, P, that a speculative attack occurs decreases.

(ii) The bound $\overline{A}(r)$, above which the policymaker adopts a peg, increases, implying that policymakers adopt pegs for a narrower range of values of A.

(iii) The equilibrium value of the policymaker's objective, V, falls.

Part (i) of Proposition 3 states that lifting restrictions on the free flow of capital induces policymakers to pursue less ambitious stabilization objectives by allowing the exchange rate to move freely within a wider band. This result is consistent with the flexibilization of exchange-rate regimes following the gradual elimination of restrictions on capital flows in the aftermath of the Bretton Woods system. Moreover, the proposition states that this reduction in transaction costs lowers, on balance, the likelihood of a currency crisis. This result reflects the operation of two opposing effects. First, as Proposition 1 shows, the two RECs shrink when t decreases. Holding the band width constant, this raises the probability of speculative attacks. This effect already appears in the literature on international financial crises (e.g., Morris and Shin 1998). But, as argued above, following the decrease in t, the band becomes wider, and this lowers, in turn, the probability, P, of speculative attacks. The analytics of these opposing effects can be seen by rewriting equation (A-23) from the appendix as:

$$\frac{\partial P}{\partial t} = -2f(\overline{\pi} + r)\frac{\partial r}{\partial t} - 2f(\overline{\pi} + r)\frac{\partial \overline{\pi}}{\partial r}\frac{\partial r}{\partial t}.$$

The first term represents the effect of an increase in a Tobin tax on the RECs for a given exchange-rate band. Because $\partial r/\partial t > 0$ (by Proposition 1), this term reduces the probability of a crisis. The second term reflects the effect of the tax increase, via its effect on the RECs, on the choice of band width. Since (as argued in Proposition 3) $\partial \overline{\pi}/\partial r < 0$, this term raises the probability of a crisis. Obviously, when the tax is reduced the signs of those two terms are interchanged. Part (i) of Proposition 3 suggests that in our model the second effect dominates, so P decreases when t is reduced.[18] Technically, this result follows because, by Assumption 1, $\partial \overline{\pi}/\partial r < -1$, i.e., when the size of the REC increases by a certain amount, the policymaker optimally chooses to reduce $\overline{\pi}$ by a larger amount.

Admittedly, our model makes specific assumptions about the policymaker's maximization problem and about the distribution of shocks in the economy. When these assumptions do not hold, the same two opposing effects on the probability of speculative attacks still operate, but the sign of their combined effect on the probability of attack may be different. Thus, the more general warranted conclusion is that, when the endogeneity of the exchange-rate regime is taken into account, an increase in the Tobin tax may increase the probability of currency attacks. Our model is an example of a case in which this happens.

Part (ii) of Proposition 3 predicts that, for symmetric distributions of fundamentals, liberalization of the capital account, as characterized by a reduction in t, should induce fewer countries to maintain pegs. It also implies that, in spite of this trend, countries with a strong preference for

[18] This result is reminiscent of the discussion in Kupiec (1996) establishing that, when general equilibrium effects are taken into consideration, a securities transaction tax does not necessarily reduce stock return volatility.

exchange-rate stability (e.g., small open economies with relatively large shares of foreign currency denominated trade and capital flows as well as emerging markets) will continue to peg even in the face of capital market liberalization. In contrast, countries with intermediate preferences for exchange-rate stability (e.g., more financially mature economies with a larger fraction of domestically denominated debt and capital flows) will move from pegs to bands. These predictions seem to be consistent with casual evidence. Two years following the 1997–1998 East Asian crisis, most emerging markets countries in that region were back on pegs (McKinnon 2001; Calvo and Reinhart 2002). On the other hand, following the EMS currency crisis at the beginning of the 1990s, the prior system of cooperative pegs was replaced by wide bands until the formation of the EMU at the beginning of 1999.

Finally, part (iii) of Proposition 3 shows that, although a decrease in t lowers the likelihood of a financial crisis, it nonetheless makes the policymaker worse off. The reason is that speculative attacks impose a constraint on the policymaker when choosing the optimal exchange-rate regime. A decrease in t strengthens the incentive to mount a speculative attack and thus makes this constraint more binding.

Importantly, the conception underlying the analysis here is that a Tobin tax as originally conceived by Tobin, is imposed only on short-term speculative trading and not on current account transactions and long-term capital flows.[19] Hence it affects short-term speculative trading, and through it government intervention, but not current account transactions and long-term capital flows, whose impact on the exchange rate is modeled by means of the exogenous stochastic variable x. For realizations of x outside the band and a given exchange-rate regime, the model captures the fact that a Tobin tax reduces speculative trading and causes the actual exchange rate to be closer on average to the boundaries of the band via the, endogenous, behavior of π.

[19] We abstract from some of the practical difficulties involved in distinguishing between short-term speculative flows and longer-term capital flows.

4.2 The Effects of Intensity of Aversion to Exchange-Rate Uncertainty

We now turn to the effects of the parameter A (the relative importance that the policymaker assigns to reduction of exchange-rate uncertainty) on the choice of regime. As argued above, in small open economies with large fractions of assets and liabilities denominated in foreign exchange, residents are more averse to nominal exchange-rate uncertainty than residents of large, relatively closed economies, whose financial assets and liabilities are more likely to be denominated in domestic currency. Hence the parameter A reflects the size of the economy and the degree to which it is open, with larger values of A being associated with smaller and more open economies.

Proposition 4 *Suppose that the policymaker's problem has a unique interior solution. Then, as A increases (the policymaker becomes more concerned with exchange rate stability):*

(i) $\bar{\pi}$ and $\underline{\pi}$ shift closer to zero, so the band becomes tighter; and

(ii) the probability, P, that a speculative attack will occur increases.

Proposition 4 states that, as the policymaker becomes more concerned with reduction of uncertainty, he sets a tighter band and allows the exchange rate to move freely only within a narrower range around the center rate.[20] Part (ii) of the proposition shows that this tightening of the band raises the likelihood of a speculative attack. This implies that, *all else equal*, policymakers in countries with larger values of A are willing to set tighter bands and face a higher likelihood of speculative attacks than policymakers in otherwise similar countries with lower values of A.

Note that as Proposition 2 shows, when A increases above $\overline{A}(r)$, the optimal band width becomes zero and so the optimal regime is a peg. On the other hand, when A falls and becomes smaller than $c'(\cdot)$, the optimal band width becomes infinite, and so the optimal regime is a free float. Given that a substantial part of international trade is invoiced in U.S. Dollars (McKinnon 1979), it is likely that policymakers of a key currency country like the United States will be less sensitive to nominal exchange-rate uncertainty and therefore have a smaller A than policymakers in

[20] This result may appear obvious at first blush. But the fact that it obtains only under unimodality (Assumption 1) suggests that such preliminary intuition is incomplete in the absence of suitable restrictions on the distribution of fundamentals.

small open economies. Therefore, our model predicts that the United States, Japan, and the Euro area should be floating, whereas Hong Kong, Panama, Estonia, Lithuania, and Bulgaria should be on either pegs, currency boards, or even full dollarization. This prediction appears to be consistent with casual observation of the exchange-rate systems chosen by those countries.

4.3 The Effects of Increased Variability in Fundamentals

Next, we examine how the exchange-rate band changes when more extreme realizations of x become more likely. This comparative statics exercise involves shifting probability mass from moderate realizations of x that do not lead to realignments to more extreme realizations that do lead to realignments.

Proposition 5 *Suppose that the policymaker's problem has a unique interior solution. Also suppose that $f(x)$ and $g(x)$ are two symmetric density functions with a mode (and a mean) at zero such that*

(i) $g(x)$ lies below $f(x)$ for all $\underline{\pi} - r < x < \bar{\pi} + r$, and

(ii) $g(\bar{\pi} + r) = f(\bar{\pi} + r)$ and $g(\underline{\pi} - r) = f(\underline{\pi} - r)$,

where $\underline{\pi}$ and $\bar{\pi}$ are the solutions to the policymaker's problem under the original density function $f(x)$ (i.e., $g(x)$ has fatter tails than $f(x)$). Then, the policymaker adopts a wider band under $g(x)$ than under $f(x)$.

Intuitively, as more extreme realizations of x become more likely (the density $f(x)$ is replaced by $g(x)$), the policymaker is more likely to incur the loss of future credibility associated with realignments. Therefore, the policymaker widens the band to offset the increase in the probability that a costly realignment will take place. In addition, as larger shocks become more likely, the policymaker also finds it optimal to shift the two RECs further away from zero in order to shift his commitment to intervene in the market to a range of shocks that are now more probable. This move benefits the policymaker's objectives by counteracting part of the increased uncertainty about the freely floating value of the exchange rate. Both factors induce a widening of the band.

4.4 The Effects of Tightness of Commitment to Maintaining the Regime

The degree of commitment to the exchange-rate regime is represented in our model by the parameter δ. Using the assumption that $f(x)$ is symmetric (in which case $E\pi = 0$) and totally differentiating equation (6) with respect to δ reveals that, in general, δ has an ambiguous effect on the optimal width of the band. On one hand, Proposition 1 implies that the width of the two RECs, r, increases as δ increases. This is because — given the width of the band — speculators are less likely to attack the band when they know that the policymaker is more likely to defend it. This reduced likelihood of attacks induces the policymaker to set a narrower band. On the other hand, as δ increases, the cost of realignments (when they occur) increases because they lead to a larger credibility loss. This effect pushes the policymaker to widen the band. Overall, then, the width of the band may either increase or decrease with δ.

Since the probability of speculative attack, P, is affected by the width of the band, the effect of δ on P is also ambiguous. For a given regime, Proposition 1 implies that P decreases with δ (since speculators are less likely to attack when they know that the policymaker is more likely to defend a given band). However, when the endogeneity of the regime is recognized, the discussion in the previous paragraph implies that this result may be reversed. In particular, when δ increases the policymaker may decide to narrow the band since he knows that, given the width of the band, he will subsequently decide to maintain the regime for a larger set of values of x. This, in turn, may increase the ex ante probability of a speculative attack. Consequently, an increase in the tightness of commitment may increase the probability of speculative attack.[21]

5 Concluding Reflections

This paper develops a framework for analyzing the interaction between the *ex ante* choice of exchange-rate regime and the probability of *ex post* currency attacks. To the best of our knowl-

[21] We also tried to characterize the optimal degree of commitment to the regime but, since the ratio of economic insights to algebra was low, this experiment is not presented. Cukierman, Kiguel and Liviatan (1992) and Flood and Marion (1999) present such an analysis for exogenously given pegs. The analysis here is more complex owing to the fact that it involves the simultaneous choice of the band width and the degree of commitment.

edge, this is the first paper that solves endogenously for the optimal regime and for the probability of currency attacks and studies their interrelation.

Our framework generates several novel predictions that are consistent with empirical evidence. First, we find that financial liberalization that lowers the transaction costs of switching between currencies induces the policymaker to adopt a more flexible exchange-rate regime. This is broadly consistent with the flexibilization of exchange-rate regimes following the gradual reductions of restrictions on capital flows in the aftermath of the Bretton Woods system (Isard 1995). Second, in our model, small open economies with substantial aversion to exchange-rate uncertainty are predicted to have narrower bands and more frequent currency attacks than large, relatively closed economies. This is broadly consistent with the fact that large economies with key currencies — such as the United States, Japan, and the Euro area — choose to float, while small open economies like Argentina (until the beginning of 2002), Thailand, and Korea choose less flexible regimes that are more susceptible to currency attacks like the 1997–1998 Southeast Asian crisis. In a wider sense, the paper suggests that a higher risk of currency attack is the price that small open economies are willing to pay for smaller exchange-rate uncertainty. Third, we show that increased variability in fundamentals generates wider bands.

Another prediction of the model, not highlighted so far, is related to the bipolar view. According to this hypothesis, in the course of globalization there has been a gradual shift away from intermediate exchange-rate regimes to either hard pegs or freely floating regimes (Fischer 2001). Globalization is expected to have two opposite effects in our model. On one hand, it lowers the cost of switching between currencies and hence facilitates speculation; this effect induces policymakers to set more flexible regimes. On the other hand, globalization increases the volume of international trade in goods and financial assets, thereby increasing the aversion to nominal exchange-rate uncertainty; this effect induces policymakers to set less flexible regimes. The second effect is likely to be large for small open economies whose currencies are not used much for either capital account or current account transaction in world markets, and to be small or even negligible for large key currency economies. Hence, the first effect is likely to be dominant in large, relatively closed blocks while the second is likely to dominate in small open economies. All else equal, the process of globalization should therefore induce relatively large currency blocks to move toward more flexible exchange-rate arrangements while pushing small open economies in the opposite direction.

Another result of our model is that a Tobin tax raises the probability of currency attacks. Although (as in existing literature) a Tobin tax reduces the probability of a currency attack for a given exchange-rate regime, the analysis also implies that the tax induces policymakers to set less flexible regimes. Hence, once the choice of an exchange-rate regime is endogenized, the tax has an additional, indirect, effect on the likelihood of a currency attack. In our model, this latter effect dominates the direct effect. Similarly, our model suggests that the effect of a larger credibility loss — following a realignment — on the probability of speculative attacks is ambiguous. For a given regime, when this credibility loss is higher, policymakers have a stronger incentive to defend the exchange-rate regime against speculative attacks, and this lowers the probability of such an event. However, once the choice of a regime is endogenized, the overall effect becomes ambiguous since ex ante, realizing that speculative attacks are less likely, policymakers may have an incentive to adopt a less flexible regime.

The model can be extended to allow for imperfect information on the part of the public about the commitment ability of policymakers. In such a framework there are two types of policymaker: a dependable type — who is identical to the one considered in this paper — and an opportunistic type, for whom the personal cost of realignment (perhaps due to a high degree of, politically motivated, positive time preference) is zero. The latter type lets the exchange rate float ex post for all realizations of fundamentals. As in Barro (1986), the probability assigned by the public to a dependable type being in office is taken as a measure of reputation. The model in this paper obtains as a particular case of the extended case when reputation is perfect. The extended analysis appears in Cukierman, Goldstein and Spiegel (2003, Section 5) and is not presented here for the sake of brevity.

An interesting implication of the extended framework is that policymakers with high reputation tend to set less flexible regimes and are less vulnerable to speculative attacks. Hong Kong's currency board is a good example. Because it has never abandoned its currency board in the past, Hong-Kong's currency board enjoys a good reputation and attracts less speculative pressure.

Another implication of the extended framework is that the width of the REC's is an increasing function of reputation, which provides an explanation for the triggers of some crises like the 1994 Mexican crisis or the 1992 flight from the French Franc following the rejection of the Maastricht Treaty by Danish voters.[22]

Although our framework captures many empirical regularities regarding exchange-rate regimes and speculative attacks, it obviously does not capture all of them. For example, as Calvo and Reinhart (2002) have shown, policymakers often intervene in exchange-rate markets even in the absence of explicit pegs or bands. We believe that an extension analyzing the desirability of implicit bands (as well as other regimes is) a promising direction for future research.[23] Another such direction is the development of a dynamic framework in which the fundamentals are changing over time and speculators can attack the currency at several points in time. The optimal policy in a dynamic context raises additional interesting issues, such as changes in the policymaker's reputation over time.

6 Appendix

Proof of Lemma 1: **(i)** We analyze the behavior of the policymaker and the speculators after the exchange rate reaches the upper bound of the band. We show that as $\varepsilon \to 0$, there exists a

[22] Prior to the Mexican crisis, Mexico maintained a peg for several years and had, therefore, good reputation. When the ruling party's presidential candidate, Colosio, was assassinated in March 1994, the Mexican Peso came under attack. The authorities defended the Peso initially but, following a substantial loss of reserves within a short period of time, allowed it to float. The extended framework provides an explanation for the crisis within a unique equilibrium framework. Prior to Colosio's assassination, fundamentals were already stretched so that, in the absence of intervention, the Peso would have depreciated. But, since reputation was high, speculators anticipated that the Mexican government would defend the peg for the existing range of realizations of x and thus refrained from attacking it. The assassination and the subsequent political instability led to an abrupt decrease in reputation, narrowing the RECs around the Mexican peg and creating a new situation in which the free market rate, x, fell outside the positive REC. It then became rational for speculators to run on the Peso and for the Mexican government not to defend it. A similar explanation can be applied to the Danish episode described in Isard (1995, p. 210).

[23] A theoretical discussion of implicit bands appears in Koren (2000). See also Bartolini and Prati (1999).

unique perfect Bayesian equilibrium in which each speculator i attacks the band if and only if θ_i is above a unique threshold $\overline{\theta}^*$. The proof for the case where the exchange rate reaches the lower bound of the band is analogous.

We start with some notation. First, suppose that $x \geq \overline{\pi}$ and let $\alpha^*(x)$ be the critical measure of speculators below which the policymaker defends the upper bound of the band when the laissez-faire rate of change in the exchange rate is x. Recalling that the policymaker defends the band if and only if $C(y, \alpha) \leq \delta$, and using the fact that $y = x - \overline{\pi}$, $\alpha^*(x)$ is defined implicitly by

$$C(x - \overline{\pi}, \alpha^*(x)) = \delta. \tag{A-1}$$

Since $C(x - \overline{\pi}, \alpha^*(x))$ increases with both arguments, $d\alpha^*(x)/dx \leq 0$.

Then, the net payoff from attacking the upper bound of the band is:

$$v(x, \alpha) = \begin{cases} (x - \overline{\pi})\, e_{-1} - t, & if \quad \alpha > \alpha^*(x), \\ -t, & if \quad \alpha \leq \alpha^*(x). \end{cases} \tag{A-2}$$

Note that $\partial v(x, \alpha)/\partial \alpha \geq 0$ because the assumption that $x \geq \overline{\pi}$ implies that the top line in (A-2) exceeds the bottom line. Moreover, noting from (A-1) that $d\alpha^*(x)/dx \leq 0$, it follows that $\partial v(x, \alpha)/\partial x \geq 0$ with a strict inequality whenever $v(x, \alpha) \geq 0$.

Let $\alpha_i(x)$ be speculator i's belief about the measure of speculators who will attack the band for each level of x. We will say that the belief $\widehat{\alpha}_i(\cdot)$ is higher than $\alpha_i(\cdot)$ if $\widehat{\alpha}_i(\cdot) \geq \alpha_i(\cdot)$ for all x with strict inequality for at least one x.

The decision of speculator i whether or not to attack $\overline{e}$ depends on the signal θ_i that speculator i observes and his belief, $\alpha_i(\cdot)$. Using (3), the net expected payoff of speculator i from attacking $\overline{e}$ is:

$$h(\theta_i, \alpha_i(\cdot)) = \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x \mid \theta_i) dx = \frac{\int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x) dx}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \tag{A-3}$$

We now establish three properties of $h(\theta_i, \alpha_i(\cdot))$:

Property 1: $h(\theta_i, \alpha_i(\cdot))$ is continuous in θ_i.

Property 2: $\widehat{\alpha}_i(\cdot) \geq \alpha_i(\cdot)$ implies that $h(\theta_i, \widehat{\alpha}_i(\cdot)) \geq h(\theta_i, \alpha_i(\cdot))$ for all θ_i.

Property 3: $\partial h(\theta_i, \alpha_i(\cdot))/\partial\theta_i \geq 0$ if $\alpha_i(\cdot)$ is nondecreasing in x with strict inequality whenever $h(\theta_i, \alpha_i(\cdot)) \geq 0$.

Property 1 follows because $F(\cdot)$ is a continuous function. Property 2 follows because $\partial v(x, \alpha)/\partial\alpha \geq 0$. To establish Property 3, note that

$$
\begin{aligned}
\frac{\partial h(\theta_i, \alpha_i(\cdot))}{\partial\theta_i} &= \frac{\left[v\left(\theta_i+\varepsilon, \alpha_i(\theta_i+\varepsilon)\right) f(\theta_i+\varepsilon) - v\left(\theta_i-\varepsilon, \alpha_i(\theta_i-\varepsilon)\right) f(\theta_i-\varepsilon)\right] \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\
&\quad - \frac{\left[f(\theta_i+\varepsilon) - f(\theta_i-\varepsilon)\right] \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} v(x, \alpha_i(x)) f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\
&= \frac{f(\theta_i+\varepsilon) \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \left[v\left(\theta_i+\varepsilon, \alpha_i(\theta_i+\varepsilon)\right) - v(x, \alpha_i(x))\right] f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\
&\quad + \frac{f(\theta_i-\varepsilon) \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \left[v(x, \alpha_i(x)) - v\left(\theta_i-\varepsilon, \alpha_i(\theta_i-\varepsilon)\right)\right] f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2}.
\end{aligned}
\tag{A-4}
$$

Recalling that $\partial v(x, \alpha)/\partial x \geq 0$ and $\partial v(x, \alpha)/\partial\alpha \geq 0$, it follows that $\partial h(\theta_i, \alpha_i(\cdot))/\partial\theta_i \geq 0$ if $\alpha_i(\cdot)$ is nondecreasing in x. Moreover, (A-3) implies that if $h(\theta_i, \alpha_i(\cdot)) \geq 0$, then there exists at least one $x \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$ for which $v(x, \alpha_i(\cdot)) > 0$ (otherwise, $h(\theta_i, \alpha_i(\cdot)) < 0$). Since $\partial v(x, \alpha)/\partial x > 0$ if $v(x, \alpha) \geq 0$, it follows in turn that $\partial v(x, \alpha)/\partial x > 0$ for at least one $x \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$. But since $\partial v(x, \alpha)/\partial x \geq 0$ with strict inequality for at least one x, (A-4) implies that $\partial h(\theta_i, \alpha_i(\cdot))/\partial\theta_i > 0$ whenever $h(\theta_i, \alpha_i(\cdot)) \geq 0$.

In equilibrium, the strategy of speculator i is to attack $\overline{e}$ if $h(\theta_i, \alpha_i(\cdot)) > 0$ and not attack it if $h(\theta_i, \alpha_i(\cdot)) < 0$. Moreover, the equilibrium belief of speculator i, $\alpha_i(\cdot)$, must be consistent with the equilibrium strategies of all other speculators (for short we will simply say that, in equilibrium, "the belief of speculator i is consistent"). To characterize the equilibrium strategies of speculators, we first show that there exists a range of sufficiently large signals for which speculators have a dominant strategy to attack $\overline{e}$ and, likewise, there exists a range of sufficiently small signals for which speculators have a dominant strategy not to attack $\overline{e}$. Then, we use an iterative process of elimination of dominated strategies to establish the existence of a unique signal, $\overline{\theta}^*$, such that speculator i attacks $\overline{e}$ if and only if $\theta_i > \overline{\theta}^*$.

Suppose that speculator i observes a signal $\theta_i > \overline{\theta}$, where $\overline{\theta}$ is defined by the equation $C\left(\left(\overline{\theta} - \overline{\pi} - \varepsilon\right), 0\right) = \delta$. Then speculator i realizes that the policymaker is surely going to exit the band. By (A-2), the net payoff from attacking $\bar{e}$ is therefore $v(x,\alpha) = (x - \bar{\pi})e_{-1} - t$, for all α. From Assumption 2, it follows that $v(x,\alpha)$ is strictly positive within the range $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. Hence, by (A-3), $h(\theta_i, \alpha_i(x)) > 0$ for all $\theta_i > \bar{\theta}$ and all $\alpha_i(x)$, implying that it is a dominant strategy for speculator i with $\theta_i > \bar{\theta}$ to attack $\bar{e}$. Similarly, if $\theta_i < \underline{\theta}$, where $\underline{\theta} \equiv \bar{\pi} + t/e_{-1} - \varepsilon$ (since we focus on the case where $\varepsilon \to 0$ and since $t > 0$, such signals are observed with a positive probability whenever $x > \bar{\pi}$), speculator i realizes that $x < \bar{\pi} + t/e_{-1}$. Consequently, even if the policymaker surely exits the band, the payoff from attacking it is negative. This implies in turn that $h(\theta_i, \alpha_i(x)) < 0$ for all $\theta_i < \underline{\theta}$ and all $\alpha_i(x)$, so it is a dominant strategy for speculator i with $\theta_i < \underline{\theta}$ not to attack.

Now, we start an iterative process of elimination of dominated strategies from $\bar{\theta}$ in order to expand the range of signals for which speculators will surely attack $\bar{e}$. To this end, let $\alpha(x,\theta)$ represent a speculator's belief regarding the measure of speculators who will attack $\bar{e}$ for each level of x, when the speculator believes that all speculators will attack $\bar{e}$ if and only if their signals are above some threshold θ. Since $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$,

$$\alpha(x,\theta) = \begin{cases} 0, & if \quad x < \theta - \varepsilon, \\ \frac{x-(\theta-\varepsilon)}{2\varepsilon}, & if \quad \theta - \varepsilon \leq x \leq \theta + \varepsilon, \\ 1, & if \quad x > \theta + \varepsilon. \end{cases} \tag{A-5}$$

The iterative process of elimination of dominated strategies works as follows. We have already established that $h(\theta_i, \alpha_i(x)) > 0$ for all $\theta_i > \bar{\theta}$ and all $\alpha_i(x)$. But since $h(\theta_i, \alpha_i(x))$ is continuous in θ_i, it follows that $h(\bar{\theta}, \alpha_i(x)) \geq 0$ for all $\alpha_i(x)$ and in particular for $\alpha_i(x) = \alpha(x, \bar{\theta})$. Thus, $h(\bar{\theta}, \alpha(x, \bar{\theta})) \geq 0$. Note that since in equilibrium the beliefs of speculators are consistent, only beliefs that are higher than or equal to $\alpha(x, \bar{\theta})$ can hold in equilibrium (because all speculators attack $\bar{e}$ when they observe signals above $\bar{\theta}$). Thus, we say that $\alpha(x, \bar{\theta})$ is the "lowest" consistent belief on α.

Let $\bar{\theta}^1$ be the value of θ_i for which $h(\theta_i, \alpha(x, \bar{\theta})) = 0$. That is, $h(\bar{\theta}^1, \alpha(x, \bar{\theta})) \equiv 0$. Note that $\bar{\theta}^1 \leq \bar{\theta}$, and that $\bar{\theta}^1$ is defined uniquely because we showed above that $h(\theta_i, \alpha_i(x))$ is strictly increasing in θ_i whenever $h(\theta_i, \alpha_i(x)) \geq 0$. Using Properties 2 and 3 and recalling that $\alpha(x, \bar{\theta})$ is the lowest consistent belief on α, it follows that $h(\theta_i, \alpha_i(x)) > 0$ for any $\theta_i > \bar{\theta}^1$ and any consistent belief $\alpha_i(x)$. Thus, in equilibrium, speculators must attack $\overline{e}$ if they observe signals above $\overline{\theta}^1$. As a result, $\alpha(x, \overline{\theta}^1)$ becomes the lowest consistent belief on $\alpha_i(x)$.

Starting from $\overline{\theta}^1$, we can now repeat the process along the following steps (these steps are similar to the ones used to establish $\overline{\theta}^1$). First, note that since $h(\overline{\theta}^1, \alpha(x, \overline{\theta})) \equiv 0$ and since $\alpha(x, \theta)$ is weakly decreasing with θ and $h(\theta_i, \alpha_i(x))$ is weakly increasing with $\alpha_i(x)$, it follows that $h(\overline{\theta}^1, \alpha(x, \overline{\theta}^1)) \geq 0$. Second, find a $\theta_i \leq \overline{\theta}^1$ for which $h(\theta_i, \alpha(x, \overline{\theta}^1)) = 0$ and denote it by $\overline{\theta}^2$. Using the same arguments as above, $\overline{\theta}^2$ is defined uniquely. Third, since $\alpha(x, \overline{\theta}^1)$ is the lowest consistent belief on $\alpha_i(x)$ and using the second and third properties of $h(\theta_i, \alpha_i(x))$, it follows that speculators must attack $\overline{e}$ if they observe signals above $\overline{\theta}^2$. The lowest possible belief on $\alpha_i(x)$ becomes $\alpha(x, \overline{\theta}^2)$.

We repeat this process over and over again (each time lowering the value of θ above which speculators will attack $\overline{e}$), until we reach a step n such that $\overline{\theta}^{n+1} = \overline{\theta}^n$, implying that the process cannot continue further. Let $\overline{\theta}^\infty$ denote the value of θ at which the process stops. (Clearly, $\overline{\theta}^\infty \leq \overline{\theta}$.) By definition, speculators will attack $\overline{e}$ if they observe signals above $\overline{\theta}^\infty$. Since $\overline{\theta}^\infty$ is the point where the process stops, it must be the case that $h(\overline{\theta}^\infty, \alpha(x, \overline{\theta}^\infty)) = 0$ (otherwise, we could find some $\theta_i < \overline{\theta}^\infty$ for which $h(\theta_i, \alpha(x, \overline{\theta}^\infty)) = 0$, meaning that the iterative process could have been continued further).

Starting a similar iterative process from $\underline{\theta}$ and following the exact same steps, we also obtain a signal $\underline{\theta}^\infty$ $(\geq \underline{\theta})$ such that speculators will never attack $\overline{e}$ if they observe signals below or at $\underline{\theta}^\infty$ (strictly speaking, at this signal the speculator is indifferent between attacking and not attacking; we break the tie by assuming that, when indifferent, the speculator chooses not to attack). At this signal, it must be the case that $h(\underline{\theta}^\infty, \alpha(x, \underline{\theta}^\infty)) = 0$. Since we proved that in equilibrium speculators attack $\overline{e}$ if they observe signals above $\overline{\theta}^\infty$ and do not attack it if they observe signals below $\underline{\theta}^\infty$, it must be the case that $\overline{\theta}^\infty \geq \underline{\theta}^\infty$.

The last step of the proof involves showing that $\overline{\theta}^\infty = \underline{\theta}^\infty$ as $\varepsilon \to 0$. First, recall that $\overline{\theta}^\infty$ is defined implicitly by $h(\overline{\theta}^\infty, \alpha(x, \overline{\theta}^\infty)) = 0$. Using (A-3) and (A-5), this equality can be written as

$$\frac{\int_{\overline{\theta}^{\infty}-\varepsilon}^{\overline{\theta}^{\infty}+\varepsilon} v\left(x, \frac{x-(\overline{\theta}^{\infty}-\varepsilon)}{2\varepsilon}\right) f(x)dx}{F\left(\overline{\theta}^{\infty}+\varepsilon\right) - F\left(\overline{\theta}^{\infty}-\varepsilon\right)} = 0. \tag{A-6}$$

Using the equality $\alpha = \left(x - (\overline{\theta}^{\infty} - \varepsilon)\right)/2\varepsilon$ to change variables in the integration, (A-6) can be written as:

$$\begin{aligned} & \frac{2\varepsilon \int_0^1 v(\overline{\theta}^{\infty} + 2\varepsilon\alpha - \varepsilon, \alpha) f(\overline{\theta}^{\infty} + 2\varepsilon\alpha - \varepsilon) d\alpha}{F\left(\overline{\theta}^{\infty}+\varepsilon\right) - F\left(\overline{\theta}^{\infty}-\varepsilon\right)} \\ = & \frac{\int_0^1 v(\overline{\theta}^{\infty} + 2\varepsilon\alpha - \varepsilon, \alpha) f(\overline{\theta}^{\infty} + 2\varepsilon\alpha - \varepsilon) d\alpha}{\frac{F\left(\overline{\theta}^{\infty}+\varepsilon\right) - F\left(\overline{\theta}^{\infty}-\varepsilon\right)}{2\varepsilon}} = 0. \end{aligned} \tag{A-7}$$

As $\varepsilon \to 0$, this equation becomes $\int_0^1 v(\overline{\theta}^{\infty}, \alpha)d\alpha = 0$ (by L'Hôpital's rule, the denominator approaches $f\left(\overline{\theta}^{\infty}\right)$ as $\varepsilon \to 0$). Likewise, as $\varepsilon \to 0$, $\underline{\theta}^{\infty}$ is defined implicitly by $\int_0^1 v(\underline{\theta}^{\infty}, \alpha)d\alpha = 0$. Now, assume by way of negation that $\overline{\theta}^{\infty} > \underline{\theta}^{\infty}$. Since $\partial v(x,\alpha)/\partial x \geq 0$ with a strict inequality when $v(x,\alpha) \geq 0$, it follows that $\int_0^1 v(\overline{\theta}^{\infty}, \alpha)d\alpha > \int_0^1 v(\underline{\theta}^{\infty}, \alpha)d\alpha$ (the strict inequality follows because $\int_0^1 v(\overline{\theta}^{\infty}, \alpha)d\alpha = 0$ implies that $v(\overline{\theta}^{\infty}, \alpha) > 0$ for at least some values of α). This inequality contradicts the fact that $\int_0^1 v(\overline{\theta}^{\infty}, \alpha)d\alpha = 0$ and $\int_0^1 v(\underline{\theta}^{\infty}, \alpha)d\alpha = 0$.

Using the notation $\overline{\theta}^* \equiv \overline{\theta}^{\infty} = \underline{\theta}^{\infty}$, we proved that as $\varepsilon \to 0$, there exists a unique threshold signal, $\overline{\theta}^*$, such that all speculators will attack $\overline{e}$ if and only if they observe signals above $\overline{\theta}^*$.

(ii) We now characterize $\overline{\theta}^*$; the characterization of $\underline{\theta}^*$ is then completely analogous. Given x and $\overline{\theta}^*$, the measure of speculators who choose to attack $\overline{e}$ is $\alpha(x, \overline{\theta}^*)$, where $\alpha(x, \overline{\theta}^*)$ is defined in (A-5). Now recall that the policymaker exits the band if $C(x - \overline{\pi}, \alpha) > \delta$. Given $\alpha(x, \overline{\theta}^*)$, a realignment takes place if and only if $C\left(x - \overline{\pi}, \alpha(x, \overline{\theta}^*)\right) > \delta$. Since $\alpha(x, \overline{\theta}^*)$ is weakly increasing in x, a realignment takes place if and only if x is above some threshold $\overline{x}(\overline{\theta}^*)$.

Note that, in a perfect Bayesian equilibrium, we cannot have $\overline{x}(\overline{\theta}^*) < \overline{\theta}^* - \varepsilon$. This can be proved by establishing a contradiction. Suppose by way of negation that $\overline{x}(\overline{\theta}^*) < \overline{\theta}^* - \varepsilon$. Then, at the signal $\overline{\theta}^*$, a speculator knows that the government abandons the band. By the definition of $\overline{\theta}^*$, the speculator is indifferent between attacking the band and not attacking it. Thus, it must be the case that when the fundamentals are at the lowest possible level given signal $\overline{\theta}^*$, the speculator gets a negative profit from attacking, i.e., $\overline{\theta}^* - \varepsilon < \overline{\pi} + t/e_{-1}$. This implies that $\overline{x}(\overline{\theta}^*) < \overline{\pi} + t/e_{-1}$. Since the policymaker is indifferent between attacking or not at $\overline{x}(\overline{\theta}^*)$, and since $C(x - \overline{\pi}, \alpha)$ is increasing in both arguments, this implies that $C\left(t/e_{-1},0\right)>\delta$, which is a contradiction to Assumption 2. Similarly, one can show that we cannot have $\overline{x}(\overline{\theta}^*)>\overline{\theta}^*+\varepsilon$. Hence, in equilibrium we must have $\overline{\theta}^*-\varepsilon\leq\overline{x}(\overline{\theta}^*)\leq\overline{\theta}^*+\varepsilon$. Let α^* be the proportion of agents who attack the currency at $\overline{x}(\overline{\theta}^*)$; this proportion is defined implicitly by:

$$C\left(\overline{x}(\overline{\theta}^*)-\overline{\pi},\alpha^*\right)=\delta. \tag{A-8}$$

Next, consider the decision problem that speculator i faces after observing the signal θ_i. Given that θ_i is drawn from the interval $[x-\varepsilon,x+\varepsilon]$, speculator i realizes that x is distributed on the interval $[\theta_i-\varepsilon,\theta_i+\varepsilon]$, and its conditional density is given by (3). But, since speculator i anticipates that the policymaker will defend $\overline{e}$ whenever $x<\overline{x}(\overline{\theta}^*)$, he expects a net payoff of $(x-\overline{\pi})e_{-1}-t$ if $x>\overline{x}(\overline{\theta}^*)$ or of $-t$ if $x<\overline{x}(\overline{\theta}^*)$. Part (i) of the lemma implies that, in equilibrium, speculators attack $\overline{e}$ if and only if they observe signals above $\overline{\theta}^*$. A speculator who observes exactly $\overline{\theta}^*$ is indifferent between attacking and not attacking. Using this indifference condition, we obtain,

$$\int_{\overline{x}(\overline{\theta}^*)}^{\overline{\theta}^*+\varepsilon}(x-\overline{\pi})e_{-1}f(x\mid\overline{\theta}^*)dx-t=0. \tag{A-9}$$

Substituting for $f(x\mid\cdot)$ from (3) into (A-9) and taking the limit as $\varepsilon\to0$, results in:

$$\lim_{\varepsilon\to0}\frac{\int_{\overline{x}(\overline{\theta}^*)}^{\overline{\theta}^*+\varepsilon}(x-\overline{\pi})e_{-1}f(x)dx}{F\left(\overline{\theta}^*+\varepsilon\right)-F\left(\overline{\theta}^*-\varepsilon\right)}=t.$$

Changing the variable of integration to α, and using (A-8), we get:

$$\lim_{\varepsilon\to0}\frac{2\varepsilon\int_{\alpha^*}^{1}(2\varepsilon\alpha+\overline{\theta}^*-\varepsilon-\overline{\pi})f(2\varepsilon\alpha+\overline{\theta}^*-\varepsilon)d\alpha}{F\left(\overline{\theta}^*+\varepsilon\right)-F\left(\overline{\theta}^*-\varepsilon\right)}=\frac{t}{e_{-1}}.$$

Using L'Hôpital's rule, and noting that $\overline{x}\left(\overline{\theta}^*\right)\to\overline{\theta}^*$ as $\varepsilon\to0$, the left-hand side of this equation becomes

$$\int_{\alpha^*}^{1}(\overline{\theta}^*-\overline{\pi})d\alpha=r\left(1-\alpha^*\right), \tag{A-10}$$

where $r\equiv\overline{\theta}^*-\overline{\pi}$. Hence,

$$r\left(1-\alpha^*\right)=\frac{t}{e_{-1}}. \tag{A-11}$$

In addition, since $\overline{x}\left(\overline{\theta}^*\right)\to\overline{\theta}^*$ as $\varepsilon\to0$, (A-8) becomes

$$C\left(r,\alpha^*\right)=\delta. \tag{A-12}$$

Substituting for α^* from (A-11) into (A-12) yields the expression that appears in the proposition and implicitly defines r. Recalling that $C(\cdot,\cdot)$ is increasing in both of its arguments, it is easy to verify from these two equations that r is increasing in t and in δ and is independent of $\overline{\pi}$. Note also that r has to be positive. This is because $\overline{\theta}^* \geq \underline{\theta}$, which means that as $\varepsilon \to 0$, $\overline{x}\left(\overline{\theta}^*\right) - \overline{\pi}$ cannot be below $\frac{t}{e-1}$.

(iii) This part follows directly from the assumption that $\varepsilon \to 0$, which implies that $\overline{x}(\overline{\theta}^*) \to \overline{\theta}^*$. **Q.E.D.**

Proof of Lemma 2: First, note that

$$\begin{aligned} E\pi &= \int_{-\infty}^{\underline{\pi}-r} xdF(x) + \int_{\underline{\pi}-r}^{\underline{\pi}} \underline{\pi}dF(x) + \int_{\underline{\pi}}^{\overline{\pi}} xdF(x) + \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi}dF(x) + \int_{\overline{\pi}+r}^{\infty} xdF(x) \quad \text{(A-13)} \\ &= -\int_{\underline{\pi}-r}^{\underline{\pi}} (x-\underline{\pi})\, dF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r} (x-\overline{\pi})\, dF(x), \end{aligned}$$

where the second equality follows because the mean of x is 0 by Assumption 3. Next, note that by Assumption 1, $\partial E\pi/\partial\underline{\pi} = \int_{\underline{\pi}-r}^{\underline{\pi}} (f(x) - f(\underline{\pi}-r))\, dx > 0$. Hence,

$$\begin{aligned} E\pi &\leq -\int_{-r}^{0} xdF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r} (x-\overline{\pi})\, dF(x) \quad \text{(A-14)} \\ &= \int_{0}^{r} xdF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r} (x-\overline{\pi})\, dF(x), \end{aligned}$$

where the equality follows from the symmetry of $f(x)$ around 0. Now, suppose that $\overline{\pi} \geq r$. Then,

$$\begin{aligned} &\int_{0}^{r} xdF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r} (x-\overline{\pi})\, dF(x) \\ <\ &\int_{0}^{r} xdF(x) + \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi}dF(x) \quad \text{(A-15)} \\ <\ &\int_{0}^{r} \overline{\pi}dF(x) + \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi}dF(x) < \overline{\pi}\int_{0}^{\overline{\pi}+r} dF(x) < \overline{\pi}, \end{aligned}$$

implying that $E\pi < \overline{\pi}$. If $\overline{\pi} < r$, then

$$\begin{aligned} &\int_{0}^{r} xdF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r} (x-\overline{\pi})\, dF(x) \\ =\ &\int_{0}^{\overline{\pi}} xdF(x) + \int_{\overline{\pi}}^{r} xdF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r} xdF(x) + \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi}dF(x) \\ <\ &\overline{\pi}\int_{0}^{\overline{\pi}+r} dF(x) + \int_{\overline{\pi}}^{r} xdF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r} xdF(x) \quad \text{(A-16)} \\ =\ &\overline{\pi}\int_{0}^{\overline{\pi}+r} dF(x) - \int_{r}^{\overline{\pi}+r} xdF(x) \\ <\ &\overline{\pi}\int_{0}^{\overline{\pi}+r} dF(x) < \overline{\pi}, \end{aligned}$$

so once again, $E\pi < \overline{\pi}$. The proof that $E\pi > \underline{\pi}$ is analogous. **Q.E.D.**

Proof of Lemma 3: Consider any arbitrary asymmetric band with $-\underline{\pi} \neq \overline{\pi}$. We will show that the policymaker's objective function attains a higher value at a symmetric band with either $(-\overline{\pi}, \overline{\pi})$ or $(\underline{\pi}, -\underline{\pi})$. To this end, we will use the property that since $f(x)$ is symmetric around 0, then for $0 < a < b$,

$$f(x) = f(-x), \quad \int_{-b}^{-a} f(x)dx = \int_{a}^{b} f(x)dx, \quad \int_{-b}^{-a} xdF(x) = -\int_{a}^{b} xdF(x) \tag{A-17}$$

Moreover, since $C(\cdot, \alpha)$ depends only on the absolute size of the disequilibrium that the policymaker tries to maintain, $c(\cdot) \equiv C(\cdot, 0)$ is symmetric as well in the sense that $c'(x - \overline{\pi}) = c'(\underline{\pi} - x)$.

Noting from (A-13) that at a symmetric band, $E\pi = 0$, and using (A-17), the policymaker's objective function, evaluated at the symmetric band $(-\overline{\pi}, \overline{\pi})$ and $(\underline{\pi}, -\overline{\pi})$ can be written as:

$$\begin{aligned} V(-\overline{\pi}, \overline{\pi}) \quad = \quad & -2A\left[\int_{0}^{\overline{\pi}} xdF(x) + \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi}dF(x) + \int_{\overline{\pi}+r}^{\infty} xdF(x)\right] \\ & -2\int_{\overline{\pi}+r}^{\infty} \delta dF(x) - 2\int_{\overline{\pi}}^{\overline{\pi}+r} c(x - \overline{\pi})dF(x), \end{aligned} \tag{A-18}$$

and

$$\begin{aligned} V(\underline{\pi}, -\underline{\pi}) \quad = \quad & 2A\left[\int_{-\infty}^{\underline{\pi}-r} xdF(x) + \int_{\underline{\pi}-r}^{\underline{\pi}} \underline{\pi}dF(x) + \int_{\underline{\pi}}^{0} xdF(x)\right] \\ & -2\int_{-\infty}^{\underline{\pi}-r} \delta dF(x) - 2\int_{\underline{\pi}-r}^{\underline{\pi}} c(\underline{\pi} - x)dF(x). \end{aligned} \tag{A-19}$$

Now, suppose that $-\underline{\pi} < \overline{\pi}$. The value of the policymaker's objective function at $(\underline{\pi}, \overline{\pi})$ is given by (5) where, due to the symmetry of $f(x)$, $E\pi > 0$. Given (A-18) and (A-19),

$$\begin{aligned} & V(-\overline{\pi}, \overline{\pi}) + V(\underline{\pi}, -\underline{\pi}) - 2V(\underline{\pi}, \overline{\pi}) \\ = \quad & -4A\left[\int_{0}^{E\pi} xdF(x) + \frac{E\pi\left(1 - 2F\left(E\pi\right)\right)}{2}\right] \\ = \quad & -4A\left[E\pi F\left(E\pi\right) - \int_{0}^{E\pi} F(x)dx + \frac{E\pi\left(1 - 2F\left(E\pi\right)\right)}{2}\right] \\ = \quad & -4A\left[\frac{E\pi}{2} - \int_{0}^{E\pi} F(x)dx\right] \\ = \quad & -4A\left[\int_{0}^{E\pi} \left(\frac{1}{2} - F(x)\right) dx\right] > 0, \end{aligned} \tag{A-20}$$

where the second equality follows from integration by parts, and the inequality follows because $f(x)$ is symmetric and $E\pi > 0$, so $F(x) > 1/2$ for all $x \in (0, E\pi]$. Equation (A-20) implies that either $V(-\overline{\pi}, \overline{\pi}) > V(\underline{\pi}, \overline{\pi})$ or $V(\underline{\pi}, -\underline{\pi}) > V(\underline{\pi}, \overline{\pi})$ or both. Hence, it is never optimal for the policymaker to select an asymmetric band with $-\underline{\pi} < \overline{\pi}$. The proof for the case where $\overline{\pi} \leq -\underline{\pi}$ is analogous. **Q.E.D**.

Conditions for an Internal Solution: Using (6), it follows that

$$\begin{aligned}
\frac{\partial V^2(\underline{\pi}, \overline{\pi})}{\partial \overline{\pi}^2} &= \int_{\overline{\pi}}^{\overline{\pi}+r} \left(A - c'(x - \overline{\pi})\right) f'(\overline{\pi} + r) dx \\
&\quad - \int_{\overline{\pi}}^{\overline{\pi}+r} c''(x - \overline{\pi}) \left(f(x) - f(\overline{\pi} + r)\right) dx \\
&\quad + \left(A - c'(0)\right) \left(f(\overline{\pi}) - f(\overline{\pi} + r)\right) + \delta f'(\overline{\pi} + r) \\
&= \int_{\overline{\pi}}^{\overline{\pi}+r} \left(A - c'(x - \overline{\pi})\right) f'(\overline{\pi} + r) dx \\
&\quad - c'(x - \overline{\pi}) \left(f(x) - f(\overline{\pi} + r)\right)\Big|_{\overline{\pi}}^{\overline{\pi}+r} + \int_{\overline{\pi}}^{\overline{\pi}+r} c'(x - \overline{\pi}) f'(x) dx \qquad \text{(A-21)} \\
&\quad + \left(A - c'(0)\right) \left(f(\overline{\pi}) - f(\overline{\pi} + r)\right) + \delta f'(\overline{\pi} + r) \\
&= \int_{\overline{\pi}}^{\overline{\pi}+r} \left(A - c'(x - \overline{\pi})\right) f'(\overline{\pi} + r) dx \\
&\quad - \int_{\overline{\pi}}^{\overline{\pi}+r} \left(A - c'(x - \overline{\pi})\right) f'(x) dx + \delta f'(\overline{\pi} + r) \\
&= \int_{\overline{\pi}}^{\overline{\pi}+r} \left(A - c'(x - \overline{\pi})\right) \left(f'(\overline{\pi} + r) - f'(x)\right) dx + \delta f'(\overline{\pi} + r),
\end{aligned}$$

where the first equality follows from integration by parts of $\int_{\overline{\pi}}^{\overline{\pi}+r} c''(x - \overline{\pi}) \left(f(x) - f(\overline{\pi} + r)\right) dx$ and the second equality follows because $A \left(f(\overline{\pi}) - f(\overline{\pi} + r)\right) = - \int_{\overline{\pi}}^{\overline{\pi}+r} A f'(x) dx$. From (A-21) it follows that $A \geq c'(y)$ for all y and $f''(x) \leq 0$, along with Assumption 1, are sufficient (but not necessary) conditions for $V(\underline{\pi}, \overline{\pi})$ to be globally concave, in which case equating the expression in (6) to zero is sufficient for a unique internal maximum.

Proof of Proposition 2: **(i)** Assumption 1 implies that $f(x) > f(\overline{\pi} + r)$ for all $x \in [\overline{\pi}, \overline{\pi} + r]$. Hence, using (6), if $A \leq c'(y)$ for all y, $\partial V(\underline{\pi}, \overline{\pi})/\partial \overline{\pi} > 0$ for all $\overline{\pi} > 0$, implying that the policymaker will push $\overline{\pi}$ all the way to ∞.

(ii) To show that $\overline{\pi} > 0$, it is sufficient to show that evaluated at $\overline{\pi} = 0$, $\partial V(\underline{\pi}, \overline{\pi})/\partial \overline{\pi} > 0$ (the policymaker will increase $\overline{\pi}$ above 0). Using (6), we obtain that

$$\left.\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}}\right|_{\overline{\pi}=0} = -\int_0^r \left(A - c'(x)\right)\left(f(x) - f(r)\right)dx + \delta f(r)$$
$$= \int_0^r \left(f(x) - f(r)\right)dx \left[\frac{\delta f(r) + \int_0^r c'(x)\left(f(x) - f(r)\right)dx}{\int_0^r \left(f(x) - f(r)\right)dx} - A\right] \quad \text{(A-22)}$$
$$= \int_0^r \left(f(x) - f(r)\right)dx \left[\overline{A}(r) - A\right],$$

where the third equality follows from integration by parts of $\int_0^r c'(x)\left(f(x) - f(r)\right)dx$ and using the fact that $c(0) = 0$. By Assumption 1, the integral term outside the brackets in the second line of (A-22) is positive. Hence, if $A < \overline{A}(r)$, the bracketed term is positive too, implying that it is optimal to set $\overline{\pi} > 0$.

To show that $\overline{\pi} < \infty$, it is sufficient to show that $V(0,0)$ is greater than $V(-\infty,\infty)$. Using (5), and the symmetry of the band, this condition holds when $\underline{A}(r) < A$.

(iii) If $V(\underline{\pi},\overline{\pi})$ is concave, a sufficient condition for $\overline{\pi} = 0$ is that, evaluated at $\overline{\pi} = 0$, $\partial V(\underline{\pi},\overline{\pi})/\partial\overline{\pi} \leq 0$ (the policymaker would not like to increase $\overline{\pi}$ above 0). Equation (A-22) ensures that the last inequality is satisfied when $A > \overline{A}(r)$. **Q.E.D.**

Proof of Proposition 3: **(i)** Proposition 1 implies that r decreases as t decreases. Straightforward differentiation of (6), using Assumption 1 and the assumption that $A \geq c'(y)$ for all y, reveals that $\bar{\pi}$ increases when r decreases. Hence a decrease in t leads to a an increase in $\bar{\pi}$ (and since $\underline{\pi} = -\bar{\pi}$, to a decrease in $\underline{\pi}$).

By Lemma 1, the probability of a speculative attack is $P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r))$. But since $\underline{\pi} = -\bar{\pi}$, $P = F(-\overline{\pi} - r) + (1 - F(\overline{\pi} + r))$. Using the fact that $f(x) = f(-x)$, yields

$$\frac{\partial P}{\partial t} = -\left(f(-\overline{\pi} - r) + f(\overline{\pi} + r)\right)\left[\frac{\partial\overline{\pi}}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial r}{\partial t}\right]$$
$$= 2f(\overline{\pi} + r)\left[-\frac{\partial\overline{\pi}}{\partial r} - 1\right]\frac{\partial r}{\partial t}. \quad \text{(A-23)}$$

By Proposition 1, $\partial r/\partial t > 0$. Hence, it is sufficient to establish that $-\partial\overline{\pi}/\partial r > 1$. Using (6) and (A-21),

$$-\frac{\partial\overline{\pi}}{\partial r} = \frac{\frac{\partial^2 V(\underline{\pi},\overline{\pi})}{\partial\overline{\pi}\partial r}}{\frac{\partial^2 V(\underline{\pi},\overline{\pi})}{\partial\overline{\pi}^2}} = \frac{\int_{\overline{\pi}}^{\overline{\pi}+r}\left(A - c'(x - \overline{\pi})\right)f'(\overline{\pi} + r)dx + \delta f'(\overline{\pi} + r)}{\int_{\overline{\pi}}^{\overline{\pi}+r}\left(A - c'(x - \overline{\pi})\right)\left(f'(\overline{\pi} + r) - f'(x)\right)dx + \delta f'(\overline{\pi} + r)}. \quad \text{(A-24)}$$

The second-order condition for an internal solution for $\overline{\pi}$ implies that the denominator of this expression is negative. Assumption 1 and $A > c'(y)$ for all y imply that the numerator is negative as well. But the denominator is equal to the numerator minus $\int_{\overline{\pi}}^{\overline{\pi}+r} (A - c'(x - \overline{\pi})) f'(x)dx$. Assumption 1 and $A > c'(y)$ for all y imply that this expression is negative. It follows that the denominator is smaller in absolute value than the numerator, implying that the expression in equation (A-24) is larger than one.

(ii) Differentiating $\overline{A}(r)$ with respect to t yields:

$$\begin{aligned} \frac{\partial \overline{A}(r)}{\partial t} &= \frac{(\delta - c(r)) f'(r) \int_0^r (f(x) - f(r)) dx + \int_0^r f'(r)dx \left(\delta f(r) - \int_0^r c(x)f'(x)dx\right)}{\left(\int_0^r (f(x) - f(r)) dx\right)^2} \frac{\partial r}{\partial t} \\ &= f'(r) \frac{(\delta - c(r)) \int_0^r f(x)dx + rc(r)f(r) - r\int_0^r c(x)f'(x)dx}{\left(\int_0^r (f(x) - f(r)) dx\right)^2} \frac{\partial r}{\partial t} < 0, \end{aligned} \quad \text{(A-25)}$$

where the inequality follows because $f'(r) < 0$ by Assumption 1, $\partial r/\partial t > 0$ by Proposition 1, and by (A-12), $\delta = C(r, \alpha^*) > C(r, 0) \equiv c(r)$. Part (iii) of Proposition 2 implies that the policymaker prefers to set a peg when $A > \overline{A}(r)$. Since $\overline{A}(r)$ falls with t, this condition is more likely to hold when t is larger.

(iii) Using (5) for the case $E\pi = 0$ and the envelope theorem, it follows that:

$$\frac{\partial V(\underline{\pi}, \overline{\pi})}{\partial t} = (rA - c(r) + \delta) \left[f(\underline{\pi} - r) + f(\overline{\pi} + r)\right] \frac{\partial r}{\partial t} > 0, \quad \text{(A-26)}$$

where the inequality follows because $\delta > c(r)$ and because by Proposition 1, $\partial r/\partial t > 0$. **Q.E.D**

Proof of Proposition 4: **(i)** The proof follows by straightforward differentiation of (6) with respect to A and by using Assumption 1 and the fact that $\underline{\pi} = -\bar{\pi}$.

(ii) By Lemma 1, the probability of a speculative attack is $P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r))$. Straightforward differentiation of P with respect to A, along with part (i) of the proposition, establish the result. **Q.E.D.**

Proof of Proposition 5: Let $\bar{\pi}^f$ be the solution to the policymaker's maximization problem when the density function is $f(x)$ and let $\bar{\pi}^g$ be the corresponding solution when the density function is $g(x)$. $\bar{\pi}^g$ is defined by (6) with $g(x)$ replacing $f(x)$. Now, let's evaluate $\partial V(\underline{\pi}, \overline{\pi})/\partial \overline{\pi}$ when the density is $g(x)$ at $\bar{\pi}^f$.

$$
\begin{aligned}
\left.\frac{\partial V(\underline{\pi},\bar{\pi})}{\partial \bar{\pi}}\right|_{\bar{\pi}=\bar{\pi}^f} &= -\int_{\bar{\pi}^f}^{\bar{\pi}^f+r}\Big(A - c'(x-\bar{\pi}^f)\Big)\Big(g(x) - g(\bar{\pi}^f+r)\Big)\,dx + \delta g(\bar{\pi}^f+r) \\
&= -\int_{\bar{\pi}^f}^{\bar{\pi}^f+r}\Big(A - c'(x-\bar{\pi}^f)\Big)\Big(g(x) - f(\bar{\pi}^f+r)\Big)\,dx + \delta f(\bar{\pi}^f+r) \qquad \text{(A-27)} \\
&> -\int_{\bar{\pi}^f}^{\bar{\pi}^f+r}\Big(A - c'(x-\bar{\pi}^f)\Big)\Big(f(x) - f(\bar{\pi}^f+r)\Big)\,dx + \delta f(\bar{\pi}^f+r) = 0,
\end{aligned}
$$

where the first equality follows since, by assumption, $g(\bar{\pi}^f+r) = f(\bar{\pi}^f+r)$. The inequality follows because $f(x)$ lies above $g(x)$ whenever $x < \bar{\pi}^f + r$, and the last equality follows from the first-order condition in (6). Since (A-27) implies that $\partial V(\underline{\pi},\bar{\pi})/\partial\bar{\pi}|_{\bar{\pi}=\bar{\pi}^f} > 0$, it follows that $\bar{\pi}^f < \bar{\pi}^g$. The analogous result for the lower bound of the band follows by recalling that, since the two distributions and the corresponding optimal bands are symmetric, $\underline{\pi}^g = -\bar{\pi}^g$ and $\underline{\pi}^f = -\bar{\pi}^f$. **Q.E.D.**

7 References

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