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# The Credibility of Monetary Announcements

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## 1. *Introduction*

Karl Brunner's interest in rules for monetary policy is well-known. Since the Federal Reserve would not follow pre-announced rules and Congress did not require them to do so, Brunner (1977) supported the efforts of Robert Weintraub and others to require the Federal Reserve to announce targets for monetary growth.<sup>1</sup> He testified in favor of targets after Congress approved Congressional Resolution 133. This resolution required the Federal Reserve to provide the public and Congress with more precise information about the monetary actions that they contemplated. The announcements have not proven to be reliable guides to actual money growth. Should they be abandoned?

This paper investigates the relationship between the credibility of monetary announcements and the structure of the policymaker's objectives when the policymaker must make announcements but is not bound by any precise rule, and is not required to make precise announcements.<sup>2</sup> The paper shows that when the policymaker has an information advantage about its own shifting objectives, and policy is discretionary, announcements will neither be completely credible, nor will they be completely disregarded. The importance given by the public to announcements in forecasting future monetary trends does not depend on the absolute degree of precision of announcements. It depends instead on the relative degree of noise in the announcements and in the control of money supply. With imprecise monetary control, relatively more precise announcements will command high credibility even if they are absolutely imprecise.

1 Announcements are (or have been) made in Germany, Japan, U.K., France, Canada, Australia and Switzerland.

2 In practice, the Federal Reserve announces a range for monetary growth.

Two measures for the credibility of announcements are proposed; the first – average credibility – is inversely related to the distance between the current announcements and the public's beliefs. The second – marginal credibility – measures the extent to which a one unit change in announced targets affects expectations. Both measures are negatively related to the relative noise in announcements and in monetary control. Average credibility is also reduced when the objectives of the monetary authority undergo large changes.<sup>3</sup> Average credibility has the attribute of a capital good that is built up and depleted gradually. The more limited is the control of money by the policymaker and the noisier his announcements, the longer it takes to build up and to deplete the „stock“ of credibility.

Since announcements have been instituted in order to provide the public and Congress with more information about future monetary policy it is important to determine whether they reduce the public's uncertainty. The paper gives an affirmative answer to this question by showing that preannouncement of monetary targets never increases the level of monetary uncertainty and usually decreases it. In this sense the requirement to disclose targets is probably useful to the public. Other results are:

- a. A Friedman (1960) type constant rate of money growth rule decreases the level of monetary uncertainty even below the level achieved under discretionary policy with preannouncements of targets.
- b. Preannouncement of targets reduces the level of monetary variability<sup>4</sup> but not to the level that could be attained with a constant planned rate of monetary growth.
- c. When the policymaker must make completely credible announcements he is induced to plan a constant rate of money growth even if he has the discretion to do otherwise. The rate he chooses is zero.
- d. The larger the degree of time preference of the policymaker the larger the monetary uncertainty experienced by the public.

Section 2 presents the model of central bank behaviour and the formation of expectations by the public. The decision rule of the policymaker and the rationality of expectations are discussed in section 3. Section 4 presents measures of credibility of announcements and relates them to some underlying

3 Credibility here refers to the degree to which announcements are believed. It is imperfect because of imprecise announcements rather than because of dynamic inconsistency of the type discussed by Kydland and Prescott (1977) and Barro and Gordon (August 1983). Further differences between those two concepts of credibility and their underlying conceptual framework are discussed in Cukierman (1985).

4 Variability and uncertainty are not necessarily identical. See for example Cukierman (1984) chapter 4, section 4.

parameters of the environment. A comparison of monetary variability and monetary uncertainty in the presence and in the absence of announcements appears in section 5. Concluding remarks follow.

## 2. *The Model*

The model in this section is a generalization of the model presented in Cukierman and Meltzer (1985) (CM in what follows). This model features a monetary authority whose relative preference for different objectives, such as economic stimulation via monetary surprises or inflation prevention, shifts in a stochastic manner over time. The shift changes the emphasis given to particular objectives when choosing money growth.

Despite the statutory independence of many central banks, their policies are partially responsive to the conflicting desires of other governmental institutions. Students of the Fed suggest that the formulation of monetary policy in the U.S. is not divorced from the general democratic process (Weintraub (1978), Kane (1980, 1982), Wooley (1984), Hetzel (1985) are examples). The Fed is partly responsive to the desires of the president (Beck (1982)) as well as to pressures from Congress – the body to which it is formally responsible.<sup>5</sup> However, because of its statutory independence, the Fed has some authority to decide whose wishes to accommodate first and by, how much.

The model has two crucial features. First, the Federal Reserve is better informed than the public about its objectives at a given time. Since the Fed's objectives are private information, the public does not learn about changes in objectives directly. Second, monetary control is imperfect, so the public cannot immediately infer the Fed's objectives by observing the rate of money growth. Persistent changes in money growth, resulting from changes in objectives, are mixed with control errors. By observing money growth, the public gradually and rationally learns about the emphasis given to different objectives of monetary policy.

The model is rational in the sense that the actual behavior that emerges is the same as the behavior on which the public relies to form expectations about future money growth. The policymaker knows how the public forecasts monetary growth and inflation, so he can calculate, up to a random shock, the

<sup>5</sup> Part of those desires are motivated by political-distributional considerations. A summary view of the political approach to central bank behavior appears in section II of Cukierman (1985).

effect of a given choice of monetary growth on surprise creation. The policymaker chooses the rate of money growth by comparing the benefits (to the policymaker) from surprise creation against the costs of higher inflation. Each period the policymaker plans to achieve a certain rate of money growth  $m_i^p$ . Actual money growth,  $m_i$ , may differ from the planned rate because control is imperfect. Specifically

$$(1) \quad m_i = m_i^p + B\varepsilon_i$$

where  $B$  is a known constant and  $\varepsilon_i$  is period's  $i$  realization of a stochastic, serially uncorrelated, normal variate with zero mean and variance  $\sigma_\varepsilon^2$ .<sup>6</sup> The variance  $B^2\sigma_\varepsilon^2$  of the noise term in (1) reflects the extent to which the operating procedures and the institutional environment prevent perfect control of money growth.

The policymaker's decision-making strategy is:

$$(2) \quad \{m_i^p, i = 0, 1, \dots\} \underset{E_{G0}}{\text{Max}} \sum_{i=0}^{\infty} \beta^i \left( e_i x_i - \frac{(m_i^p)^2}{2} \right)$$

$$(3) \quad e_i \equiv m_i - E[m_i | I_i]$$

$$(4) \quad x_i = A + p_i \quad A > 0$$

$$(5) \quad p_i = \rho p_{i-1} + v_i, \quad 0 < \rho < 1$$

where  $v$  is a serially uncorrelated normal variate with zero mean, variance  $\sigma_v^2$  and is distributed independently of the control error  $\varepsilon$ . Here  $e_i$  is the unanticipated rate of money growth in period  $i$ ,  $I_i$  the information available to the public at the beginning of period  $i$ , and  $E[m_i | I_i]$  the public's forecast of  $m_i$  given the information set  $I_i$ .  $\beta$  is the policymaker's subjective discount factor, and  $E_{G0}$  is a conditional expected value operator that is conditioned on the information available to government in period 0 including a direct observation on  $x_0$ . Ceteris paribus the policymaker prefers lower to higher inflation.

$x_i$  is a random shift parameter which determines the shifts in the policymaker's

6 There is no loss of generality in specifying the multiplicative constant in front of  $\varepsilon_i$  as  $B$  rather than 1 since the variance  $\sigma_\varepsilon^2$  can always be adjusted so as to yield any desired value for the variance  $B^2\sigma_\varepsilon^2$  of the noise term in (1). The precise value of  $B$  in terms of the underlying parameters of the model is determined later from the requirement that expectations are rational.

objectives between economic stimulation achieved through surprise creation and inflation. The higher  $x_i$  the more willing is the policymaker to trade higher inflation for more stimulation. Equations (4) and (5) specify the stochastic behavior of the shift parameter  $x_i$  and indicate that the policymaker's objectives exhibit a certain degree of persistence which depends on the size of  $A$  and  $p$ .  $x_i$  is normally positive<sup>7</sup> reflecting the view that unanticipated monetary growth stimulates employment and output and that the policymaker prefers, *ceteris paribus*, more to less stimulation.

House Concurrent Resolution 133, and later the Humphrey-Hawkins Act, require the Federal Reserve to announce planned rates of growth for principal monetary aggregates. The purpose of this legislation is to provide the public and Congress with more precise information about the particular monetary actions contemplated by the monetary authority. This information has not proven to be a reliable forecast. Even a cursory look at the „Record of Policy Actions of the Federal Open Market Committee“<sup>8</sup> shows that actual rates of monetary growth have deviated substantially from the preannounced rates. The main novelty of this paper is that it extends the model of CM to the case in which the policymaker must make announcements of future targets but is not required to make completely accurate announcements. We assume that at the beginning of each period the policymaker makes a noisy announcement,

$$(6) \quad m_i^a = m_i^p + B\eta_i$$

where  $\eta_i$  is a serially uncorrelated normal variate with zero mean, variance  $\sigma_\eta^2$  and is distributed independently of the monetary control error.<sup>9</sup>

Preannouncement gives the public an additional source of information about future monetary growth. The optimal predictor incorporates the information that the public gains from the announcement and, as in CM, the information obtained from the history of money growth. A rational policymaker must use the new information structure faced by the public when planning future money growth. To obtain a rational expectations solution for the model in the

7 The distributional assumptions on  $v$  imply that  $x \sim N(A, \sigma_v^2/(1-p^2))$  so that if  $A \sqrt{(1-p^2)}/\sigma_v$  is sufficiently large the probability that  $x$  will have a negative realization can be driven as close to zero as desired. For example when this ratio is 3 the probability is 0.0027. To avoid negative values,  $A$  has to be sufficiently large compared to  $\sigma_v/\sqrt{1-p^2}$ .

8 The „Record“ appears in the Federal Reserve Bulletin within two months after the meeting of the Federal Open Market Committee. Brunner and Meltzer (1983) summarize the record.

9 Our formulation reflects the procedures in the U.S. where the Federal Reserve decides on the content of the announcement and chooses to announce a range rather than a point target. Again there is no loss of generality in specifying  $B$  as the constant in front of  $\eta_i$  for the reason given in footnote 6.

presence of announcements, we extend our earlier model to include announcements. As before, we postulate that the policymaker's strategy is given by

$$(7) \quad m_i^p = B_0 A + B p_i,$$

where  $B_0$  and  $B$  are constants to be determined by the requirement of rational expectations.

The public knows the structure of the policymakers decisions regarding  $m_i^p$ , given by (7), and the relationship between announced and planned monetary growth in (6). The public observes only actual and announced monetary growth. At the beginning of period  $i$  the public knows the history of past rates of money growth up to and including period  $i-1$  and the history of announcements up to and including the announcement,  $m_i^a$ , made at the beginning of period  $i$ .  $I_i$  denotes this information set.

Substituting (7) into (1) and (6), we have

$$(a) \quad m_i = B_0 A + B (p_i + \epsilon_i) = B_0 A + B y_i$$

$$(8) \quad (b) \quad m_i^a = B_0 A + B (p_i + \eta_i) = B_0 A + B z_i$$

$$(c) \quad z_i \equiv p_i + \eta_i; \quad y_i \equiv p_i + \epsilon_i$$

Since  $B_0$ ,  $A$  and  $B$  are known parameters, observations on  $m_i$  and  $m_i^a$  amount to observations on  $y_i$  and  $z_i$  respectively. Note that it pays the public to use announcements to improve its forecast of the stochastic persistent component of the policymaker's objectives.

It follows, using (8a) that the public's rational expectation of  $m_i$  is,

$$(9) \quad E[m_i | I_i] = B_0 A + B E[p_i | I_i] = \\ = B_0 A + B E[p_i | z_i, z_{i-1}, \dots, y_{i-1}, y_{i-2}, \dots]$$

Part 1 of the appendix shows that

$$(10) \quad (a) \quad E[p_i | I_i] = \frac{(\rho - \delta)(\sigma_\epsilon^2 + \sigma_\eta^2)}{\rho \sigma_\epsilon^2 + \delta \sigma_\eta^2} \left\{ \sum_{j=1}^{\infty} \delta^j (\theta y_{i-j} + (1-\theta)z_{i-j}) + (1-\theta)z_i \right\}$$

$$(b) \quad \delta \equiv \frac{1}{2} \left( \frac{1+r}{\rho} + \rho \right) - \sqrt{\frac{1}{4} \left( \frac{1+r}{\rho} + \rho \right)^2 - 1}$$

$$(c) \quad r \equiv \frac{\sigma_v^2}{\sigma_\varepsilon^2} \left( 1 + \frac{\sigma_\varepsilon^2}{\sigma_\eta^2} \right) = \frac{\sigma_v^2}{\sigma_\varepsilon^2} + \frac{\sigma_v^2}{\sigma_\eta^2}$$

$$(d) \quad \theta \equiv \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}$$

Substituting (10a) into (9) and using (8)

$$(11) \quad E[m_i | I_i] = \frac{(\rho - \delta)(1 - \theta)}{\delta + (\rho - \delta)(1 - \theta)} m_i^a + \frac{\delta}{\delta + (\rho - \delta)(1 - \theta)} \sum_{j=0}^{\infty} \delta^j [(1 - \rho)\bar{m}^P + (\rho - \delta)(\theta m_{i-1-j} + (1 - \theta)m_{i-1-j}^a)]$$

where

$$\bar{m}^P \equiv B_0^A$$

is the mean rate of money growth.

Since  $\delta$  has the same form in  $\rho$  and  $r$  as  $\lambda$  in equation (10b) of CM, we can apply our previous result. (The solution for  $r$  differs in the two cases, since implicitly  $\sigma_\eta^2$  is infinity in CM). It follows that  $0 \leq \delta \leq 1$  and that  $\rho - \delta \geq 0$ . The optimal predictor is a weighted average of the current announcement and the past history of announcements and monetary growth (including  $\bar{m}^P$ )—the two terms in (11)—with weights that sum to unity. The weight placed on each term depends on  $\sigma_\eta^2$ . Noisy announcements, large  $\sigma_\eta^2$ , reduce the usefulness of announcements, so the public pays less attention to announcements. In the limit as  $\sigma_\eta^2 \rightarrow \infty$ ,  $\theta \rightarrow 1$ ; there is no information in the announcements, and they are ignored. (The optimal predictor in (11) reduces to equation (11a) of CM) At the other extreme,  $\sigma_\eta^2 \rightarrow 0$ ,<sup>10</sup> announcements are completely accurate statements of planned money growth, and the optimal predictor,  $E[m_i | I_i] = m_i^a$ . The current announcement is fully credible. This remains true even if  $\sigma_\varepsilon^2$  is relatively large and monetary control is relatively poor.

10 As  $\sigma_\eta^2 \rightarrow 0$ ,  $\theta \rightarrow 0$ ,  $r \rightarrow \infty$  and  $\delta \rightarrow 0$ .



Between the extreme values of  $\sigma_\eta^2$ ,  $\sigma_\eta^2$  and  $\sigma_\varepsilon^2$  determine the weights given to announcements and to the past history of monetary growth. In general, the noisier signal gets a smaller weight.<sup>11</sup>

### 3. The Policymaker's Decision Rule and Proof of the Rationality of Expectations

The policymaker knows that the public forms expectations according to (11), and he uses this information when choosing planned monetary growth. Substituting (11) into (3), substituting the resulting expression into (2), using (12) and rearranging, the maximization problem of the policymaker becomes

$$(13) \quad \text{Max}_{\{m_i^p, i=0,1,\dots\}} E_{G0} \left[ \sum_{i=0}^{\infty} \beta^i \left[ x_i (m_i^p + B\varepsilon_i - \frac{(1-\rho)\delta}{(1-\delta)(\rho-\delta)(1-\theta)+\delta} B_0 A \right. \right. \\ \left. \left. - \frac{(\rho-\delta)(1-\theta)}{(\rho-\delta)(1-\theta)+\delta} (m_i^p + B\eta_i) - \frac{\delta(\rho-\delta)}{(\rho-\delta)(1-\theta)+\delta} \sum_{j=0}^{\infty} \delta^j (m_{i-1-j}^p + B(\varepsilon_{i-1-j} + \eta_{i-1-j})) \right) \right. \\ \left. - \frac{(m_i^p)^2}{2} \right]$$

The stochastic Euler equations necessary for an internal maximum of this problem are<sup>12</sup>

$$(14) \quad \left( 1 - \frac{(\rho-\delta)(1-\theta)}{(\rho-\delta)(1-\theta)+\delta} \right) x_i - \frac{\delta(\rho-\delta)}{(\rho-\delta)(1-\theta)+\delta} E_{Gi} (Bx_{i+1} + B^2\delta x_{i+2} + \dots) \\ - m_i^p = 0 \quad i = 0, 1, \dots$$

CM show (equation (14)) that

$$(15) \quad E_{Gi} x_{i+j} = \rho^j x_i + (1-\rho^j)A, \quad j \geq 0$$

Substituting (15) into (14) and using (4) this expression reduces after a considerable amount of algebra to (16). (See part 2 of the appendix).

11 The sign of the partial derivative of the coefficient of  $m_i^p$  with respect to  $\sigma_\eta^2$  is opposite to that of  $\frac{\partial \delta / \partial \sigma_\eta^2 + \delta \partial \theta / \partial \sigma_\eta^2}{\sigma_\eta^2}$ . Since both  $\delta$  and  $\theta$  are increasing in  $\sigma_\eta^2$  the weight given to the current announcement is monotonically decreasing in  $\sigma_\eta^2$ .

12 Since all the terms in (14) are finite, the transversality condition is satisfied for any  $\beta < 1$ .

$$(16) \quad m_i^p = \frac{\delta(1-\rho\beta)(1 + \frac{\sigma_\epsilon^2}{\sigma_\eta^2})}{(1-\delta\beta)(\delta + \rho \frac{\sigma_\epsilon^2}{\sigma_\eta^2})} A + \frac{(1-\beta\rho^2)\delta(1 + \frac{\sigma_\epsilon^2}{\sigma_\eta^2})}{(1-\rho\beta\delta)(\delta + \rho \frac{\sigma_\epsilon^2}{\sigma_\eta^2})} p_i.$$

With  $B_0$  as the coefficient of  $A$  and  $B$  the coefficient of  $p_i$ , this is the self fulfilling solution. (Compare (7) and (16)).

When there is no information in the announcement ( $\sigma_\eta^2 \rightarrow \infty$ ) the decision strategy in (16) reduces to the decision strategy in the absence of announcements.<sup>13</sup> Hence both the optimal predictor and the policymaker's decision rule in the presence of announcements are generalizations of their respective counterparts in the economy with no announcements. Formally, therefore, the economy with no announcements can be viewed as a particular case (with  $\sigma_\eta^2 \rightarrow \infty$ ) of the economy with announcements.

From equation (8a) the unconditional mean and variance of actual money growth are given respectively by

$$(17) \quad \begin{aligned} (a) \quad E m_i &= B_0 A \\ (b) \quad V(m_i) &= (B)^2 (\sigma_p^2 + \sigma_\epsilon^2) \end{aligned}$$

$B_0$  is positive,<sup>14</sup> so the average rate of monetary expansion and the average rate of inflation are positive. The average rates increase with the government's preference for economic stimulation relative to its dislike of monetary growth and inflation. Also for a given variance of the monetary control error, the variance of monetary growth is, as in CM, a decreasing function of  $\beta$ .<sup>15</sup> The more a policymaker cares about the present relative to the future, the more he increases the variability of monetary growth, independently of whether he makes or does not make announcements.

#### 4. The Credibility of Announcements

To this point, we have not given a precise definition of credibility. In this section, we use the optimal predictor in (11) to define two measures of

13 As  $\sigma_\eta^2 \rightarrow \infty$ ,  $\delta$  tends to  $\lambda$  in CM,  $B_0$  and  $B$  become identical to  $B_0$  and  $B$  in CM and the decision strategy in (16) reduces to that in equation (15) of CM. In addition the optimal predictor in (11) reduces to equation (11a) of CM.

14  $B_0$  is positive for any  $\beta < 1$  since  $0 \geq \delta \geq 1$ .

15 The proof is qualitatively similar to the proof of proposition 2b in CM.

credibility, average and marginal credibility, and we analyze the determinants of credibility.

Average credibility measures the extent to which the public's expectations of future monetary growth deviate from the current announcement. The smaller this deviation, the larger average credibility. When  $m_i^a = E(m_i|I_i)$ , average credibility is perfect.

$$(18) \quad \text{Average Credibility} \equiv AC \equiv - |m_i^a - E(m_i|I_i)|$$

Substituting (11) into (18) and rearranging

$$(19) \quad AC = \frac{\delta}{\delta + (\rho - \delta)(1 - \theta)} |m_i^a - m_i^*|$$

where

$$(20) \quad m_i^* \equiv \sum_{j=0}^{\infty} \delta^j \{ (1 - \rho) \bar{m}^p + (\rho - \delta)(\theta m_{i-1-j} + (1 - \theta) m_{i-1-j}^a) \} .$$

$m_i^*$  summarizes all the information that individuals have about future money growth before they get the current announcement. Average credibility is low when the current announcement is far away from  $m_i^*$ . The distance between  $m_i^a$  and  $m_i^*$  rises with the difference between current and past announcements and with the difference between  $m_i^a$  and the mean rate of money growth  $\bar{m}^p$ . Average credibility is reduced both during periods with large changes in announcements and when announcements differ markedly from average experience. Further, for any given divergence between  $m_i^a$  and  $m_i^*$  average credibility is lower the larger the coefficient

$$C \equiv \frac{\delta}{\delta + (\rho - \delta)(1 - \theta)}$$

in equation (19). This coefficient is, ceteris paribus, an increasing function of  $\delta$ .<sup>16</sup> Analysis of  $C$  leads to the following proposition about average credibility.

*Proposition 1:* For a given divergence between  $m_i^a$  and  $m_i^*$  average credibility is lower:

- a. the lower is the precision of announcements (the larger  $(B \sigma_\eta)^2$ );
- b. the lower equi-proportionally is the precision of both announcements and monetary control (the higher are  $(B \sigma_\eta)^2$  and  $(B \sigma_\epsilon)^2$  for a given  $\sigma_\eta^2 | \sigma_\epsilon^2$ );

16 The sign of its partial derivative with respect to  $\delta$  is the same as the sign of  $\rho(1 - \theta) > 0$ .

c. the lower is the variance of the innovation in the policymaker's objectives,  $\sigma_v^2$ .<sup>17</sup>

Proof: a. Increases in  $\sigma_\eta^2$  increase  $\delta$  and reduce  $(1-\theta)$ , so  $C$  increases with  $\sigma_\eta^2$ . Further, the variance,  $(B \sigma_\eta)^2$ , of the noise in the announcement is also an increasing function of  $\sigma_\eta^2$ . This can be seen by noting that from (16)  $B$  depends on  $\sigma_\eta^2$  both directly and through the (positive) dependence of  $\delta$  on  $\sigma_\eta^2$ . Since both the direct effect of  $\sigma_\eta^2$  and the effect of  $\delta$  on  $B$  are positive it follows that an increase in  $\sigma_\eta^2$  is associated with a decrease in the precision of announcements and lower average credibility.

b. An equiproportional increase in  $\sigma_\epsilon^2$  and  $\sigma_\eta^2$  raises  $C$  and is equivalent to an equiproportional increase in  $(B \sigma_\epsilon)^2$  and  $(B \sigma_\eta)^2$ . In part 3 of the appendix, we show that  $(B \sigma_\epsilon)^2$  is an increasing function of  $\sigma_\epsilon^2$ .

Part c follows directly from the fact that  $C$  increases with  $\delta$  and  $\delta$  decreases with  $\sigma_v^2$ . Q.E.D.

When monetary control is loose and announcements are highly noisy a shift to new rates of monetary growth does not generate immediate credibility. In particular a change in governmental objectives toward less inflation will be recognized only gradually even if government announces the new policy. As a result the period of learning and the consequent lull in economic activity is lengthened; the cost of disinflation increases. A shift to more inflationary objectives also takes more time to be recognized when credibility is low, so the period of economic stimulation increases.

When monetary control is tight and announcements precise, the credibility of new objectives is quickly established. Note that when either monetary control is perfect ( $\sigma_\epsilon^2 = 0$ ) or announcements fully precise ( $\sigma_\eta^2 = 0$ ),  $\delta = 0$  and average credibility is perfect. Hence perfect credibility can be established either way. However, uncertainty faced by the public is larger, when the second method is used.

Average credibility focuses on the difference between the current announcement and belief. One may also be interested in the ability of the current announcement to affect expectations. A useful measure of this ability is marginal credibility.

$$\text{Marginal credibility} \equiv MC = \frac{\partial E[m_i | I_i]}{\partial m_i^a}$$

From equation (11),

$$MC = \frac{(\rho - \delta)(1 - \theta)}{\delta + (\rho - \delta)(1 - \theta)}$$

<sup>17</sup> The results in part c of the proposition has to be interpreted with care since it also reflects changes in the noisiness of monetary control and of announcements.

Considerations similar to those which led to proposition 1 show that marginal credibility falls with any increase in  $(B \sigma_\eta)^2$ , with any equiproportional increase in  $(B \sigma_\eta)^2$  and  $(B \sigma_\epsilon)^2$ , and any decrease in  $\sigma_v^2$ . Marginal credibility is perfect when  $MC = 1$  and is non existent when  $MC = 0$ . The first limiting case is attained for either  $\sigma_\epsilon^2 \rightarrow 0$  or  $\sigma_\eta^2 \rightarrow 0$ . The second is attained when both  $\sigma_\eta^2$  and  $\sigma_\epsilon^2$  tend to infinity.

Before closing this section, we note that imperfect credibility arises here without any dynamic inconsistency of the type considered by Kydland and Prescott (1977, p. 475). The basis for imperfect credibility here is the policymaker's advantage over the public that is due to shifting objectives, noisy control and noisy announcements. The policymaker knows his stochastically changing objectives, but the public does not. The best the public can do is to form expectations, allowing for this noise, and use all the information available each period to infer current and future money growth. Announcements are not fully credible because they are noisy despite the fact that the policymaker is known to be following dynamically consistent discretionary policies.

### 5. *Comparison of Uncertainty and Variability under Alternative Monetary Arrangements*

In this section we compare the uncertainty faced by the public under alternative monetary arrangements when monetary control is imperfect. The variance of the monetary control error is the same in each regime. We compare three alternative monetary arrangements: policy is discretionary, and there are no announcements of targets; policy is discretionary but the policymaker is required to announce a target, as in the U.S. since 1975; and there is a rule of the type proposed by Friedman (1960) that requires constant monetary growth. Uncertainty is measured by the variance of the one period ahead error in predicting monetary growth.

We begin by considering the effect of announcements. The variances of the one period ahead errors in the presence and in the absence of announced targets are respectively

$$(21) \quad \begin{aligned} (a) \quad & v^*(e) \equiv E\{m_i - E\{m_i | I_i^*\}\}^2 \\ (b) \quad & v(e) = E\{m_i - E\{m_i | I_i\}\}^2 \end{aligned}$$

where  $I_i^*$  is the public's information set in the presence of announcements and  $I_i$  its information set in the absence of announcements. In this section we will identify parameters and variables in the economy with announcements by a starred superscript. Thus  $\varepsilon_i^*$  and  $\varepsilon_i$  are the noisy components of monetary control in the presence and in the absence of announcements respectively. Similarly  $B^*$  now denotes the coefficient of  $p_i$  in equation (8) while the symbol  $B$  (previously used to denote this coefficient) will be reserved for the same coefficient in the absence of announcements. The reason two different white noise processes are used to describe noisy control procedures is that we want to compare the level of uncertainty in economies with and without announcements for the *same* variance of monetary control error. To that end equation (1) is respecified as

$$(1') \quad \begin{aligned} (a) \quad m_i &= m_i^p + B^* \varepsilon_i^* && \text{with announcements.} \\ (b) \quad m_i &= m_i^p + B \varepsilon_i && \text{without announcements.} \end{aligned}$$

The condition that the variance of the monetary control error is the same with and without announcements implies

$$(22) \quad v_\psi \equiv [B^*(\sigma_{\varepsilon^*}^2, \sigma_\eta^2)]^2 \sigma_{\varepsilon^*}^2 = [B(\sigma_\varepsilon^2)]^2 \sigma_\varepsilon^2.$$

The explicit form of  $B^*(\cdot)$  is given by the coefficient of  $p_i$  in equation (16), and the explicit form of  $B(\cdot)$  is (from equation (16) of CM)

$$(23) \quad B = \frac{1-\beta\rho^2}{1-\beta\rho\lambda}$$

where  $\lambda$  is the value to which  $\delta$  tends when  $\sigma_\eta^2$  tends to infinity. Substituting (8a) and (9) into (21a) and equations (7) and (9) from CM into (21b) and rearranging we obtain expressions for the variance of unanticipated monetary growth with and without announcements.<sup>18</sup>

$$(24) \quad \begin{aligned} (a) \quad v^*(e) &= (B^*)^2 E[\varepsilon_i^* + p_i - E[p_i | z_i, z_{i-1}, \dots, y_{i-1}^*, y_{i-2}^*, \dots]]^2 \\ (b) \quad v(e) &= B^2 E[\varepsilon_i + p_i - E[p_i | y_{i-1}, y_{i-2}, \dots]]^2 \end{aligned}$$

where

<sup>18</sup> Instead of reestablishing all of the results used in the case of no announcements, we refer the reader to the relevant sections of our earlier paper, Cukierman & Meltzer (1985).

$$(25) \quad y_i^* \equiv p_i + \varepsilon_i^*$$

Noting that  $\varepsilon_i$  and  $\varepsilon_i^*$  are distributed independently of their respective lagged values and of  $\eta$  and using the fact that the conditional expected values of  $p_i$  are minimum mean square error estimators of  $p_i$ , equations (24a) and (24b) can be rewritten

$$(26) \quad (a) \quad v^*(e) = v_\psi + [B^*(\sigma_{\varepsilon^*}^2, \sigma_\eta^2)]^2 \min_{\{a_j^*, c_j\}} E[p_i - \sum_{j=1}^{\infty} a_j^* y_{i-j}^* - \sum_{j=0}^{\infty} c_j z_{i-j}]^2,$$

$$(b) \quad v(e) = v_\psi + [B(\sigma_c^2)]^2 \min_{\{a_j\}} E[p_i - \sum_{j=1}^{\infty} a_j y_{i-j}]^2.$$

Here  $a_j^*$  and  $c_j$  are respectively the weights of  $y_{i-j}^*$  and  $z_{i-j}$  in the expression for the optimal predictor of  $p_i$  (see equation A1 of the appendix) and  $a_j$  is the weight of  $y_{i-j}$  when this predictor is based solely on observations of  $y$ . (See appendix 1 of CM)

Passing  $B^*(\bullet)$  and  $B(\bullet)$  in equations (26a) and (26b) to the right of the min operators, redefining variables and rearranging, we obtain

$$(27) \quad (a) \quad v^*(e) = v_\psi + \min_{\{a_j^*, c_j\}} E[q_i^* - \sum_{j=1}^{\infty} a_j^* (q_{i-j}^* + \psi_{i-j}^*) - \sum_{j=0}^{\infty} c_j (q_{i-j}^* + \phi_{i-j})]^2,$$

$$(b) \quad v(e) = v_\psi + \min_{\{a_j\}} E[q_i - \sum_{j=1}^{\infty} a_j (q_{i-j} + \psi_{i-j})]^2.$$

where

$$(28) \quad q_j^* \equiv B^* p_j, \quad \psi_j^* \equiv B^* \varepsilon_j^*, \quad \phi_j \equiv B^* \eta_j$$

$$q_j \equiv B p_j, \quad \psi_j \equiv B \varepsilon_j \quad \text{for all } j$$

The definitions in (28) in conjunction with (5) imply

$$(29) \quad E q_i = E q_i^* = 0, \quad \sigma_q^2 = B^2 \sigma_p^2, \quad \sigma_{q^*}^2 = (B^*)^2 \sigma_p^2$$

$$E q_i q_{i-j} = \rho^j \sigma_q^2, \quad E q_i^* q_{i-j}^* = \rho^j \sigma_{q^*}^2 \quad \text{for all } i \text{ and } j$$

where  $\sigma_q^2$  and  $\sigma_{q^*}^2$  are the variances of  $q$  and  $q^*$  respectively.

Expanding the square terms on the right hand sides of equations (27a) and (27b), using (29) and the fact that  $\psi_j^*$ ,  $\psi_j$  and  $\Phi_j$  are all mutually and serially uncorrelated, equations (27) can be rewritten

$$V^*(e) = V_\psi + \min_{\{a_j^*, c_j\}} \left\{ \sum_{j=1}^{\infty} (a_j^*)^2 V_\psi + \sum_{j=0}^{\infty} c_j^2 V_\phi \right.$$

(30a)

$$\left. + ((1-c_0)^2 - 2(1-c_0) \sum_{j=1}^{\infty} (a_j^* + c_j) \rho^j + \sum_{j=1}^{\infty} \sum_{t=1}^{\infty} (a_j^* + c_t)(a_t^* + c_t) \rho^{|j-t|}) \sigma_{q^*}^2 \right\}$$

$$V(e) = V_\psi + \min_{\{a_j\}} \left\{ \sum_{j=1}^{\infty} a_j^2 V_\psi \right.$$

(30b)

$$\left. + (1 - 2 \sum_{j=1}^{\infty} a_j \rho^j + \sum_{j=1}^{\infty} \sum_{t=1}^{\infty} a_j a_t \rho^{|j-t|}) \sigma_q^2 \right\}$$

where  $V_\phi$  is the variance of  $\Phi_j$ . If  $\sigma_{q^*}^2$  and  $\sigma_q^2$  are equal the minimum on the right hand side of (30a) is no larger than the minimum on the right hand side of (30b). That is

$$(31) \quad V^*(e, \sigma_{q^*}^2) \leq V(e, \sigma_q^2) \equiv V(e)$$

The reason is that the value of the minimum in (30b) can always be attained in (30a) by setting the minimizers in (30a) at the values  $a_j^* = a_j$ ,  $j = 1, 2, \dots$  and  $c_j = 0$ ,  $j \geq 0$ . But in fact

$$(32) \quad \sigma_{q^*}^2 = [B^*(\sigma_{\epsilon^*}^2, \sigma_\eta^2)]^2 \sigma_p^2 < [B(\sigma_\epsilon^2)]^2 \sigma_p^2 = \sigma_q^2$$

because  $B^*(\cdot) < B(\cdot)$ .<sup>19</sup> By the envelope theorem the total effect of a change in  $\sigma_{q^*}^2$  on  $V^*(e)$  is equal to the direct effect of  $\sigma_{q^*}^2$  on  $V^*(e)$  evaluated at the minimizing values of  $\{a_j^*, c_j\}$ . This partial derivative is given in turn by the coefficient of  $\sigma_{q^*}^2$  in (30a) evaluated at the minimum. For any  $\sigma_{q^*}^2$ , and for  $\sigma_{q^*}^2 = \sigma_q^2$  in particular, this coefficient is positive provided  $\rho < 1$ .<sup>20</sup> It follows from this observation and (32) that

<sup>19</sup> See part 4 of the appendix for a proof.

<sup>20</sup> The proof appears in part 5 of the appendix.



$$(33) \quad v^*(e) \equiv v^*(e, \sigma_{q^*}^2) < v^*(e, \sigma_q^2)$$

This together with (31) implies that preannouncement lowers the variance of the forecast errors, that is

$$(34) \quad v^*(e) < v(e) .$$

Friedman (1960) proposes a rule requiring a constant growth of money. The policymaker in our model cannot control money growth without error. A rule for constant, pre-announced money growth sets a constant value of  $m_1^p = m_1^a$  but does not make  $m_1$  constant because of the control error. The variance of the one period ahead forecast error is, in this case, due entirely to imperfect control of the money supply and is given by  $V_\psi$  in (22). It follows from this observation in conjunction with (27) and (34) that

$$(35) \quad v_\psi < v^*(e) < v(e) .$$

Equation (35) is the main result of this section. It states that, for an exogenously given variance of the monetary control error, announcements reduce uncertainty and that uncertainty can be further reduced if the policymaker follows a Friedman type rule. This result suggests, that as long as  $\sigma_\eta^2$  is not infinite the requirement to preannounce targets is not meaningless. When the policymaker behaves in a discretionary manner, mandatory announcements of targets reduce uncertainty for the public. Abandoning discretion by adopting Friedman's rule further reduces uncertainty.

It seems plausible that, for a given quality of monetary control, the level of monetary uncertainty imposed on the public rises with the policymaker's time preference. The following proposition makes this notion more precise.

*Proposition 2:* Given the variance,  $V_\psi$ , of the monetary control error the variance of the mean forecast error  $V(e)$  is larger the lower is  $\beta$ .

*Proof:* Partially differentiating the explicit expression for  $B(\cdot)$  with respect to  $\beta$  in (23) shows that the derivative is negative. To maintain the fixity of  $V_\psi$  in (22) when  $\beta$  increases,  $\sigma_\epsilon^2$  has to increase also since  $V_\psi = [B(\sigma_\epsilon^2, \beta)\sigma_\epsilon^2]^2$  is an increasing function of  $\sigma_\epsilon^2$ . With a higher  $\sigma_\epsilon^2$  and the same  $V_\psi$ ,  $B(\sigma_\epsilon^2, \beta)$  is lower than before the increase in  $\beta$ . Equation (29) shows that  $\sigma_q^2$  is lower the higher is  $\beta$ . By the envelope theorem the sign of the total effect of a change in  $\sigma_q^2$  on  $V(e)$  is the same as the sign of the coefficient of  $\sigma_q^2$  in (30b) evaluated at the minimizing values of  $\{a_j\}$ . It is shown in part 5 of the appendix that this

coefficient is positive for any  $\rho < 1$ . Hence an increase in  $\beta$ , by lowering  $\sigma_q^2$ , decreases  $V(e)$ .<sup>21</sup>

Equations (30a) and (30b) also imply that the higher the variance of the monetary control error,  $V_\psi$ , the higher the level of monetary uncertainty imposed on the public both in the presence and in the absence of pre-announcements. The level of uncertainty in the presence of announcements is also larger the larger the noisiness of those announcements as measured by  $V_\phi$ . Both results are direct consequences of the envelope theorem and of the fact that the coefficients of  $V_\psi$  and  $V_\phi$  in equations (30a) and (30b) are positive.

We close the discussion of uncertainty by relating the reduction of uncertainty achieved by Friedman's rule to the credibility problem. The reduced level of uncertainty achieved by a rule requiring a constant planned rate of money growth can be achieved under discretion and announcements if the noise component of those announcements has zero variance i.e., if  $V_\phi = \sigma_\eta^2 = 0$ . We showed above that in this case (11) implies that  $E[m_i | I_i^*] = m_i^a$ . From (6),  $m_i^a = m_i^p$ . The forecast error is  $m_i - E[m_i | I_i^*] = \psi_i^*$  with variance  $V_\psi$ . Hence the level of uncertainty can be reduced to that of Friedman's rule even if planned money growth is not required to be constant provided announcements have perfect credibility. But when  $V_\phi = 0$  the public gets advance perfect information about any changes in the plans of the policymaker and his information advantage disappears. There is no benefit to the policymaker from planning any money growth rate other than zero. Perfectly credible announcements induce the policymaker to adhere to a Friedman type rule even when he has the discretion not to do so.

The variability of monetary growth under the three monetary arrangements has the same ordering as uncertainty. Friedman's monetary rule generates the lowest variability; discretionary policy without announcements generates the most. By comparing (18) in CM with, (17) and (22)

$$\begin{aligned}
 (a) \quad & V(m_i) = V_\psi + [B(\sigma_\epsilon^2)]^2 \sigma_p^2 \\
 (36) \quad & \\
 (b) \quad & V^*(m_i) = V_\psi + [B^*(\sigma_{\epsilon^*}^2, \sigma_\eta^2)]^2 \left| \frac{\delta(1 + (\sigma_{\epsilon^*}^2 / \sigma_\eta^2))}{\delta + \rho (\sigma_{\epsilon^*}^2 / \sigma_\eta^2)} \right|^2 \sigma_p^2
 \end{aligned}$$

21 It is likely that the lower  $\beta$  for given values of the noise variances  $V_\psi$  and  $V_\phi$  the higher the variance of the mean forecast error in the presence of preannouncements. We have not proved this, however.

Under Friedman's rule  $m_1^e$  is constant, so the variance of monetary growth with such a rule is  $V_\psi$ .  $V_\psi$  is lower than the variances in (36). Further, for a given variance of the control error, announcements reduce the variance of monetary growth. To see this note that  $\rho \geq \delta$  implies

$$\delta \left( 1 + \frac{\sigma_{\epsilon^*}^2}{\sigma_\eta^2} \right) / \left( \delta + \rho \frac{\sigma_{\epsilon^*}^2}{\sigma_\eta^2} \right) \leq 1$$

This together with the fact<sup>22</sup> that  $B^*(\sigma_{\epsilon^*}^2, \sigma_\eta^2) < B(\sigma_\epsilon^2)$  implies, using (36), that  $V(m_i) > V^*(m_i)$ . The positive implication of this discussion is that, other things the same, preannouncements reduce the variability of monetary growth.

## 6. Concluding Comments

This paper develops a positive theory of credibility for a monetary system in which control is imperfect and announced targets are inaccurate. The level of credibility varies inversely with the difference between the announcement and the public's expectations of money growth. Credibility is perfect when the announced and the expected rates of monetary growth are identical.

This concept of credibility seems appropriate for countries like the U.S. where policymakers operate in a discretionary manner but are required to make announcements about the growth of monetary aggregates. In contrast, Taylor (1982) and Barro and Gordon (July 1983) focus on a somewhat different concept of credibility. They study the conditions that lead people to believe that the government has abandoned discretion in favor of a rule mandating a zero rate of money growth.

Our analysis suggests that the legal requirement to announce targets does not generate immediate credibility if announcements are noisy. In such cases credibility of new policies is established slowly. However, given the precision of monetary control, preannouncements almost always reduce the level of monetary uncertainty faced by the public.

The credibility of newly instituted disinflationary policies also depends on the quality of monetary control. With tight control, a few periods of determined

<sup>22</sup> See part 4 of the appendix.

slowdown in the rate of monetary expansion, accompanied by accurate announcements of the new policy, suffice to convince the public that money growth is permanently lower. As a consequence, expectations of inflation fall quickly. Unexpected rates of monetary growth remain negative for a relatively brief period, and the accompanying unemployment is relatively small. In this case a „cold turkey“ disinflationary policy is preferable to „gradualism“, since a large decrease in monetary expansion, pre-announced to the public, generates credibility relatively quickly. If the policymaker has poor control of money growth and is inaccurate in his announcements, disinflationary policy takes substantially longer to become credible, however. The interim period of unemployment is longer and unemployment is larger. The costs of disinflation are higher. A gradual approach, that permits the public to adjust anticipations and credibility, seems preferable in these circumstances.

Fischer (1984) provides estimates of the costs of disinflation. His analysis suggests that those costs are quite sensitive to the way expectations adjust. Our paper relates this critical speed of adjustment to some underlying factors like the quality of monetary control, the quality of monetary announcements and the discount factor of the policymaker. Fischer's analysis in conjunction with ours creates a link between the costs of disinflation on one hand and the quality of monetary control and the degree of time preference of the policymaker on the other.

In Cukierman and Meltzer (1985) we analyzed the circumstances under which the policymaker will choose imprecise control procedures even if perfect control is technically feasible. Given such a choice any additional information about the current objectives of the policymaker is potentially valuable to the public. Preannouncement of monetary targets is one type of additional, imprecise but not meaningless, information. Statements made by Fed officials about their view of the current state of the economy and related issues are another. The analysis in this paper may be viewed more generally as applying to any kind of signal issued by the central bank about its future monetary policy besides past money growth. With this wider interpretation in mind, the paper implies that the lower the precision of monetary control the more attention will be paid to other signals of planned monetary growth like public statements, rumors and personalities. In such circumstances any public appearance by a high ranking Fed official receives wide press coverage. With very precise monetary control, on the other hand, past money growth is a sufficient statistic for future plans and the pronouncements of central bank officials are given little attention.

## Appendix

### 1. Derivation of the optimal predictors in equations (10) of the text.

Since  $p_t$ ,  $m_t$ , and  $m_t^a$  are all normally distributed the expected value of  $p_t$  conditioned on  $I_t$  is a linear function with fixed coefficients of the observations on  $I_t$ . That is

$$E[p_t | I_t] = \sum_{i=1}^{\infty} a_i y_{t-i} + \sum_{i=0}^{\infty} c_i z_{t-i} \quad (A1)$$

where

$$I_t \equiv \{m_{t-1}, m_{t-2}, \dots, m_t^a, m_{t-1}^a, \dots\} = \{y_{t-1}, y_{t-2}, \dots, z_{t-1}, z_{t-2}, \dots\}$$

and

$$y_t \equiv p_t + \epsilon_t, \quad (A2)$$

are respecifications of  $I_t$  and of equation (8c) that are introduced in anticipation of the discussion in section V (see equation (25) and the discussion that precedes it). Here  $\epsilon_t$  is a white noise process with variance  $\sigma_\epsilon^2$ .

Since the conditional expected value in (A1) is also the point estimate of  $p_t$  which minimizes the mean square error around this estimate it follows that  $\{a_i\}_{i=1}^{\infty}$ ,  $\{c_i\}_{i=0}^{\infty}$  are to be chosen so as to minimize

$$Q \equiv E \left[ p_t - \sum_{i=1}^{\infty} a_i y_{t-i} - \sum_{i=0}^{\infty} c_i z_{t-i} \right]^2$$

Substituting (A2) and (8c) into this expression for  $Q$  using the fact that  $\epsilon$ ,  $\eta$  and  $v$  are mutually independent and passing the expectation operator through,  $Q$  can be rewritten

$$Q = [(1-c_0)^2 + ((1-c_0)^\rho - (a_1+c_1))^2 + ((1-c_0)^{\rho-2} - \rho(a_1+c_1) - (a_2+c_2))^2 + \dots + \quad (A3)$$

$$((1-c_0)^\rho)^{i-1} - \rho(a_1+c_1) - \dots - (a_i+c_i))^2 + \dots] \sigma_v^2 + \sum_{i=1}^{\infty} a_i^2 \sigma_\epsilon^2 + \sum_{i=0}^{\infty} c_i^2 \sigma_\eta^2$$

The necessary first order conditions for an extremum of Q are<sup>23</sup>;

$$\frac{\partial Q}{\partial c_0} = -2[1-c_0 + \rho((1-c_0)^\rho - (a_1+c_1))+\dots] \quad (A4-1)$$

$$+ \rho^i ((1-c_0)^\rho - \rho^{i-1}(a_1+c_1) - \dots - (a_i+c_i))+\dots] \sigma_v^2 + 2c_0 \sigma_n^2 = 0$$

$$\frac{\partial Q}{\partial c_i} = -2[(((1-c_0)^\rho - \rho^{i-1}(a_1+c_1) - \dots - (a_i+c_i)) \quad (A4-2)$$

$$+ \rho((1-c_0)^\rho - \rho^{i-1}(a_1+c_1) - \dots - (a_{i+1}+c_{i+1})) + \dots] \sigma_v^2 + 2c_i \sigma_n^2 = 0$$

$i \geq 1.$

$$\frac{\partial Q}{\partial a_i} = -2[(((1-c_0)^\rho - \rho^{i-1}(a_1+c_1) - \dots - (a_i+c_i)) \quad (A4-3)$$

$$+ \rho((1-c_0)^\rho - \rho^{i-1}(a_1+c_1) - \dots - (a_{i+1}+c_{i+1})) + \dots] \sigma_v^2 + 2a_i \sigma_\epsilon^2 = 0$$

$i \geq 1$

Leading (A4-2) by one period, multiplying by  $\rho$  and subtracting (A4-2) from the resulting expression

$$[(1-c_0)^\rho - \rho^{i-1}(a_1+c_1) - \dots - (a_i+c_i)] \sigma_v^2 + (\rho c_{i+1} - c_i) \sigma_n^2 = 0 \quad i \geq 1 \quad (A5)$$

Multiplying (A5) by  $\rho$ , subtracting the resulting expression from (A5) led by one period and rearranging

$$\rho \sigma_n^2 c_{i+2} - (\sigma_v^2 + (1+\rho^2) \sigma_n^2) c_{i+1} - \rho \sigma_n^2 a_{i+1} + \rho \sigma_n^2 c_i = 0 \quad i \geq 1 \quad (A6)$$

Rearranging (A4-2) and (A4-3), equating their common terms and rearranging again

$$a_i = \frac{\sigma_n^2}{\sigma_\epsilon^2} c_i \quad i \geq 1 \quad (A7)$$

<sup>23</sup>Since Q is a quadratic this extremum is a minimum.

Substituting (A7) into (A6) and dividing the resulting expression by  $\rho \sigma_n^2$

$$c_{i+2} - \left\{ \frac{1+r}{\rho} + \rho \right\} c_{i+1} + c_i = 0 \quad i \geq 1 \quad (\text{A8})$$

where

$$r \equiv \frac{\sigma_v^2}{\sigma_\epsilon^2} \left( 1 + \frac{\sigma_\epsilon^2}{\sigma_n^2} \right) \quad (\text{A9})$$

(A8) is a second order homogeneous difference equation whose general solution is

$$c_i = K \delta^i \quad i \geq 1 \quad (\text{A10})$$

where K is a constant to be determined by initial conditions and  $\delta$  is the root of the quadratic

$$u^2 - \left( \frac{1+r}{\rho} + \rho \right) u + 1 = 0 \quad (\text{A11})$$

The roots of this equation are given by

$$u_{1,2} = \frac{1}{2} \left( \frac{1+r}{\rho} + \rho \right) \pm \sqrt{\left\{ \frac{1}{2} \left( \frac{1+r}{\rho} + \rho \right) \right\}^2 - 1} \quad (\text{A12})$$

The positive root in (A12) is larger than one and the negative root is bounded between zero and one. Thus  $c_i$  does not diverge only if the smaller root is substituted for  $\delta$  in (A10). Since  $c_i$  has to yield a minimum for Q it cannot diverge. Hence

$$\delta = \frac{1}{2} \left( \frac{1+r}{\rho} + \rho \right) - \sqrt{\left\{ \frac{1}{2} \left( \frac{1+r}{\rho} + \rho \right) \right\}^2 - 1} \quad (\text{A13})$$

Substituting (A10) into (A7)

$$a_i = K \frac{\sigma_n^2}{\sigma_\epsilon^2} \delta^i \quad i \geq 1 \quad (\text{A14})$$

K is determined from the initial conditions. Substitute (A10) and (A14) into (A5) for the case  $i = 1$  and rearrange to get

$$\rho \sigma_v^2 (1 - c_0) - \sigma_\eta^2 (1 + r) K \delta + \rho \sigma_\eta^2 K \delta^2 = 0 \quad (\text{A15})$$

Specializing (A4-2) to the case  $i = 1$ , multiplying by  $\rho$ , subtracting (A4-1) from the resulting expression and rearranging

$$c_0 = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\eta^2} + \frac{\sigma_\eta^2}{\sigma_v^2 + \sigma_\eta^2} \rho c_1 \quad (\text{A16})$$

Substituting (A16) into (A15), using (A10) and rearranging

$$K = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\eta^2 (1 - \rho \delta)} \quad (\text{A17})$$

Multiplying numerator and denominator of (A17) by  $(\sigma_\epsilon^2 + \sigma_\eta^2) / \sigma_\eta^2 \sigma_\epsilon^2$  and noting the definition of  $r$  in (A9)

$$K = \frac{r}{r + 1 - \rho \delta + \frac{\sigma_\eta^2}{\sigma_\epsilon^2} (1 - \rho \delta)} \quad (\text{A17-1})$$

Since  $\delta$  is a root of (A11) it satisfies this equation, i.e.

$$\delta^2 - \left( \frac{1+r}{\rho} + \rho \right) \delta + 1 = 0 \quad (\text{A18})$$

solving for  $r$  from (A18), substituting into (A17-1) and rearranging it follows that

$$K = \frac{\sigma_\epsilon^2 (\rho - \delta)}{\rho \sigma_\epsilon^2 + \delta \sigma_\eta^2} \quad (\text{A17-2})$$

Substituting  $K$  from (A17-2) into (A10) and (A14)



$$c_i = \frac{(\rho-\delta)(\sigma_\epsilon^2 + \sigma_\eta^2)}{\rho\sigma_\epsilon^2 + \delta\sigma_\eta^2} (1-\theta)\delta^i \quad (\text{A19-1})$$

$$i \geq 1$$

$$a_i = \frac{(\rho-\delta)(\sigma_\epsilon^2 + \sigma_\eta^2)}{\rho\sigma_\epsilon^2 + \delta\sigma_\eta^2} \theta\delta^i \quad (\text{A19-2})$$

where

$$\theta \equiv \frac{\sigma_\eta^2}{\sigma_\epsilon^2 + \sigma_\eta^2}$$

Substituting (A17) into (A10) specializing to the case  $i = 1$ , substituting the resulting expression into (A16) and rearranging

$$c_0 = \frac{\sigma_v^2}{\sigma_v^2 + (1-\rho\delta)\sigma_\eta^2} = \frac{(\rho-\delta)(\sigma_\epsilon^2 + \sigma_\eta^2)}{\rho\sigma_\epsilon^2 + \delta\sigma_\eta^2} (1-\theta). \quad (\text{A20})$$

The second equality in (A20) follows by noting that  $K$  is given by both (A17) and (A17-2).

The optimal predictor follows by inserting equations (A19) and (A20) into (A1) and is given by

$$E[p_t | I_t] \equiv \frac{(\rho-\delta)(\sigma_\epsilon^2 + \sigma_\eta^2)}{\rho\sigma_\epsilon^2 + \delta\sigma_\eta^2} \left\{ \sum_{j=1}^{\infty} \delta^j (\theta y_{t-j} + (1-\theta)z_{t-j}) + (1-\theta)z_t \right\}. \quad (\text{A21})$$

## 2. Derivation of Equation (16)

Substituting (4) and (15) into (14) and rearranging the resulting expression we obtain

$$m_i^p = \frac{\delta}{(\rho-\delta)(1-\theta)+\delta} [A+p_i - (\rho-\delta)(\rho\beta+(\rho\beta)^2\delta + (\rho\beta)^3\delta^2+\dots)(A+p_i) - (\rho-\delta)\{(\beta+\beta^2\delta + \beta^3\delta^2+\dots)-(\rho\beta+(\rho\beta)^2\delta + (\rho\beta)^3\delta^2+\dots)\}A] \quad (A22)$$

Noting that

$$(\rho-\delta)(1-\theta)+\delta = \frac{\delta\sigma_\eta^2 + \rho\sigma_\epsilon^2}{\sigma_\eta^2 + \sigma_\epsilon^2},$$

summing the geometric progressions in (A22) and collecting terms

$$m_i^p = \frac{\delta(\sigma_\eta^2 + \sigma_\epsilon^2)}{\sigma_\eta^2 + \sigma_\epsilon^2} \left[ \left(1 - \frac{(\rho-\delta)\beta}{1-\beta\delta}\right) A + \left(1 - \frac{(\rho-\delta)\rho\beta}{1-\rho\beta\delta}\right) p_i \right] \quad (A23)$$

(A23) can be rewritten

$$m_i^p = \frac{\delta(1-\rho\beta)(\sigma_\eta^2 + \sigma_\epsilon^2)}{(1-\delta\beta)(\delta\sigma_\eta^2 + \rho\sigma_\epsilon^2)} A + \frac{\delta(1-\beta\rho^2)(\sigma_\eta^2 + \sigma_\epsilon^2)}{(1-\rho\beta\delta)(\delta\sigma_\eta^2 + \rho\sigma_\epsilon^2)} p_i \quad (A24)$$

Equation (16) in the text follows by dividing both numerators and denominators in (A24) by  $\sigma_\eta^2$ .

3. Proof that  $[B(\sigma_\epsilon^2, \sigma_\eta^2)]^2 \sigma_\epsilon^2$  is increasing in  $\sigma_\epsilon^2$ .

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The coefficient of  $p_i$  in (A24) is equal to  $(B(\sigma_\epsilon^2, \sigma_\eta^2))^2$  and can be rewritten

$$D_1^2 D_2^2 \sigma_\epsilon^2$$

where

$$D_1 \equiv \frac{1-\beta\rho^2}{1-\beta\rho\delta}; \quad D_2 \equiv \frac{\delta(\sigma_\eta^2 + \sigma_\epsilon^2)}{\delta\sigma_\eta^2 + \rho\sigma_\epsilon^2}$$

Since  $D_1$  is positive and increasing in  $\delta$  which in turn is increasing in  $\sigma_\epsilon^2$  it follows that  $D_1^2$  is an increasing function of  $\sigma_\epsilon^2$ . The partial derivative of  $D_2$  with respect to  $\delta$  is

$$\frac{\partial D_2}{\partial \delta} = \frac{(\sigma_\eta^2 + \sigma_\epsilon^2)\rho \sigma_\epsilon^2}{[\delta \sigma_\eta^2 + \rho \sigma_\epsilon^2]^2}$$

which is also positive. Hence  $D_2^2$  is also an increasing function of  $\sigma_\epsilon^2$  through the effect of the latter on  $\delta$ . Finally, the partial derivative of  $D_2^2 \sigma_\epsilon^2$  with respect to  $\sigma_\epsilon^2$ , holding  $\delta$  constant, is of the same sign as

$$\rho \sigma_\epsilon^4 + \delta \sigma_\eta^4 + \sigma_\epsilon^2 \sigma_\eta^2 (3\delta - \rho) \quad (A25)$$

By using the explicit expression for  $\delta$  from (10b) it can be shown that the condition  $3\delta - \rho > 0$  is equivalent to the condition

$$2\left(\frac{1+r}{\rho}\right)^2 + r + 2 > 0$$

which is always satisfied. Hence (A25) is positive too. It follows that  $[B(\bullet) \sigma_\epsilon]^2$  is an increasing function of  $\sigma_\epsilon^2$ .

#### 4. Proof that $B^*(\sigma_{\epsilon^*}^2, \sigma_\eta^2) < B(\sigma_\epsilon^2)$

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To make the notation consistent with section V of the text, we now place a "\*" on  $B$ ,  $\sigma_\epsilon^2$ ,  $a$  and  $r$  when we refer to the case of announcements. These same symbols without asterisks now refer to the case without announcements analyzed in Cukierman and Meltzer (1985).

From equations (23) and (16)

$$B(\bullet) = \frac{1-\beta\rho^2}{1-\beta\rho\lambda} \quad ; \quad B^*(\bullet) = \frac{1-\beta\rho^2}{1-\beta\rho\delta} \cdot \frac{\delta(1+a^*)}{\delta + \rho a^*} \quad (A26)$$

where  $a^* \equiv \sigma_{\epsilon^*}^2 / \sigma_\eta^2$ .

Since  $\rho \geq \lambda$ ,  $\delta(1+a^*) / (\delta + \rho a^*) \leq 1$ . Let  $\tilde{\delta}(\sigma_\epsilon^2, \sigma_\eta^2)$  and  $\tilde{r}^*(\sigma_\epsilon^2, \sigma_\eta^2)$  be

respectively the values assigned to  $\delta$  and  $r^*$  by equations (10) when  $\sigma_\epsilon^2$  (without announcements) is substituted for  $\sigma_{\epsilon^*}^2$  (with announcements). Note (using (10c) in CM and (10c) of this paper) that for any

$$\sigma_\eta^2 < \infty$$

$$\tilde{r}^*(\sigma_\epsilon^2, \sigma_\eta^2) > r$$

Since  $\tilde{\delta}(\cdot)$  is a decreasing function of  $\tilde{r}^*$  it follows that

$$\tilde{\delta}(\sigma_\epsilon^2, \sigma_\eta^2) < \lambda$$

This together with (A26) and the fact that  $\delta(1+a^*)/(\delta+\rho a^*) \leq 1$  imply

$$\left[ \frac{1-\beta\rho^2}{1-\beta\rho\tilde{\delta}(\sigma_\epsilon^2, \sigma_\eta^2)} \frac{\delta(1+a^*)}{\delta+\rho a^*} \right]^2 \sigma_\epsilon^2 < B^2 \sigma_\epsilon^2 \equiv V_\psi \quad (A27)$$

In order to maintain the equality in (22) the left hand side of (A27) has to be increased by an appropriate change of  $\sigma_\epsilon^2$ . Since the left hand side of (A27) is increasing in  $\sigma_\epsilon^2$  (see part 3 of appendix)  $\sigma_{\epsilon^*}^2$  that is defined

by (22) is larger than  $\sigma_\epsilon^2$ . It follows from (22) that

$$B^*(\sigma_{\epsilon^*}^2, \sigma_\eta^2) < B(\sigma_\epsilon^2)$$

5. Proof that the coefficients of  $\sigma_{\epsilon^*}^2$  and  $\sigma_q^2$  in equations

(30a) and (30b) are positive at the minimum for any  $\rho < 1$ .

The difference between the minimization problem in (26a) and in (27a) is only that in the latter equation the constant  $B^*$  has been absorbed into the minimization problem. Hence the minimizing values for the problem in (27a) are the same as those for the problem in (26a) for which the minimizing values of  $\{a_j^*\}$  and  $\{c_j\}$  have been derived in part 1 of the appendix and are given by

$$a_j^* = \frac{(\rho - \delta)\theta}{\rho(1-\theta) + \delta\theta} \delta^j, \quad j = 1, 2, \dots; \quad c_j = \frac{(\rho - \delta)(1-\theta)}{\rho(1-\theta) + \delta\theta} \delta^j, \quad j = 0, 1, \dots \quad (A28)$$

(This value of  $a^*$  is called  $a$  in part I of the appendix. The use of the asterisk is consistent with the notation in section V of the text)

Substituting (A28) into the coefficient of  $\sigma_{q^*}^2$  in (30a) and using repeatedly the formula for the summation of infinite geometric progressions we obtain after a considerable amount of algebra the following expression for the effect of  $\sigma_{q^*}^2$  on  $V^*(e)$

$$\frac{\partial V^*(e)}{\partial \sigma_{q^*}^2} = \frac{\delta^2(1-\rho^2)}{[\rho(1-\theta) + \delta\theta]^2(1-\delta^2)}$$

This expression is unambiguously positive for any  $\rho < 1$ . Since the proof holds for any positive value of  $\sigma_{q^*}^2$  including in particular all the range between  $\sigma_{q^*}^2$  and  $\sigma_q^2$  it follows that

$$\frac{\partial V^*(e)}{\partial \sigma_{q^*}^2} > 0$$

in all the range  $[\sigma_{q^*}^2, \sigma_q^2]$ .

The proof that  $\partial V(e)/\partial \sigma_q^2 > 0$  for all  $\rho < 1$  proceeds along analogous lines.

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