The choice of exchange rate regime and speculative attacks*

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Abstract

We develop a framework for studying the choice of exchange rate regime in an open economy where the local currency is vulnerable to speculative attacks. The framework makes it possible to study, for the first time, the strategic interaction between the ex ante choice of regime and the likelihhod of ex post currency attacks. The optimal regime is determined by a policymaker who trades off the loss from nominal exchange rate uncertainty, against the cost of maintaining a given regime. This cost in turn increases with the fraction of speculators who attack the local currency. Searching for the optimal regime within the class of exchange rate bands, we show that the optimal regime is either a peg (a zero-width band), a free float (an infinite-width band), or a non degenerate band of finite width. Our framework generates several novel predictions and shows that when the endogeneity of the exchange rate regime is recognized explicitly, conventional wisdom may be reversed. For instance a Tobin tax, designed to reduce

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the likelihood of speculative attacks by restricting international liquidity, induces policymakers to set less flexible regimes, and may, thereby, actually raise the likelihood of speculative attacks.

1 Introduction

The literature on speculative attacks and currency crises can be broadly classified into first-generation models (e.g., Krugman, 1979 and Flood and Garber, 1984) and second-generation models (e.g., Obstfeld, 1994 and 1996; Velasco, 1997; and Morris and Shin, 1998). Recent surveys by Flood and Marion (1999) and Jeanne (2000) suggest that the main difference between the two generations of models is that in first-generation models, the policies that ultimately lead to the collapse of fixed exchange rate regimes are specified exogenously, while in second-generation models, policymakers play an active role in deciding whether or not to defend the currency against a speculative attack. In other words, second-generation models endogenize the policymakers' response to a speculative attack. As Jeanne (2000, p. 5-6) points out, this evolution of the literature is similar to "the general evolution of thought in macroeconomics, in which government policy also evolved from being included as an exogenous variable in macroeconomic models to being explicitly modeled."

Although second-generation models explicitly model the policymakers' (ex post) response to speculative attacks, the initial (ex ante) choice of the exchange rate regime (typically a peg), is treated in this literature as exogenous. As a result, the interdependence between ex post currency attacks and the ex ante choice of exchange rate regime is ignored in this literature. A different line of literature that focuses on optimal exchange rate regimes (e.g., Helpman and Razin, 1982, and Devereux and Engel, 1999) also ignores this effect by abstracting from the possibility of speculative attacks.[1]

This paper takes a first step towards filling this gap by developing a model in which both the *ex ante exchange rate regime* and the *probability of ex post currency attacks* are determined endogenously. The model has three stages: In the first stage, prior to the realization of a stochastic shock to the freely floating exchange rate (the "fundamental" in the model), the policymaker chooses the exchange rate regime. In the second stage, after the realization of fundamentals, speculators decide whether or not to attack the exchange rate regime. Finally, in the third stage, the policymaker decides whether to defend the regime or abandon it. Thus, relative to second-generation models, our model explicitly examines the ex ante choice of the exchange rate regime.

[1] A recent exception is a paper by Guembel and Sussman (2002) which studies the choice of exchange rate regime in the presence of speculative trading. Their model, however, does not deal with currency crises, as it assumes that policymakers are always fully committed to the exchange rate regime. Somewhat related is a recent paper by Jeanne and Rose (2002), which analyzes the effect of the exchange rate regime on noise trading. However, they do not analyze the interaction between speculative trading and the abandonment of preannounced exchange rate regimes.

This makes it possible to rigorously examine, for the first time, the strategic interaction between the ex ante choice of regime and the probability of ex post currency attacks.

In order to model speculative attacks, we use the framework recently developed by Morris and Shin (1998) where each speculator observes a slightly noisy signal about the fundamentals of the economy, so that the fundamentals are not common knowledge among speculators. Besides making a step towards realism, this framework has the advantage of eliminating multiple equilibria of the type that arise in second generation models with common knowledge.[2] In our context, this implies that the fundamentals of the economy uniquely determine whether a currency attack will or will not occur. This uniqueness result is important since it establishes an unambiguous relation between the choice of exchange rate regime and the likelihood of currency attacks.[3]

In general, characterizing the best exchange rate regime is an extremely hard problem since the best regime may have an infinite number of arbitrary features. The difficulty is compounded by the fact that the exchange rate regime affects, in turn, the strategic behavior of speculators vis-a-vis the policymaker and vis-a-vis each other. We therefore limit the search for the "best" regime to the class of explicit exchange rate bands. This class of regimes is characterized by two parameters: the upper and the lower bounds of the band. The policymaker allows the exchange rate to move freely within these bounds but commits to intervene in the market and prevent the exchange rate from moving outside the band. Although the class of bands does not exhaust all possible varieties of exchange rate regimes, it is, nonetheless, rather broad and includes as special cases the two most commonly analyzed regimes: pegs (zero-width bands) and free floats (infinitely wide bands).[4,5] Our approach allows us to conveniently characterize the best regime in the presence

[2] Morris and Shin (2002) stress that the multiplicity of equilibria in second generation models is a consequence of two simplifying assumptions: (i) the fundamentals of the economy are common knowledge, and (ii) in equilibrium, speculators are certain about each other's behavior. Once these assumptions are relaxed, a unique equilibrium may obtain.

[3] The uniqueness result was first established by Carlsson and van Damme (1993) who refer to games in which each player observes a different signal about the state of nature as global games. Recently, the global games framework has been applied to study other issues that are related to currency crises, such as the effects of transparency (Heinemann and Illing, 2002) and interest rate policy (Angeletos, Hellwig, and Pavan, 2002). A similar framework has also been applied in other contexts (see, for example, Goldstein and Pauzner (2000) for an application to bank runs). For an excellent survey, which addresses both applications and theoretical extensions (such as inclusion of public signals in the global games framework), see Morris and Shin (2001).

[4] Interestingly, Garber and Svensson (1995) note that "...fixed exchange rate regimes in the real world typically have explicit finite bands within which exchange rate are allowed to fluctuate".

[5] Intermediate regimes (bands of positive but finite width) have been adopted during the nineties by a good number

of potential currency attacks within a substantially larger class of regimes than usually considered.

In order to focus on the main novelty of the paper, which is the strategic interaction between the ex ante choice of exchange rate regime and the probability of ex post speculative attacks, we model some of the underlying macroeconomic structure in a reduced form manner.[6] A basic premise of our framework is that exporters and importers, as well as borrowers and lenders in foreign currency denominated financial assets, dislike uncertainty about the level of the nominal exchange rate and that policymakers internalize at least part of this aversion. This premise is consistent with recent empirical findings by Calvo and Reinhart (2002). To reduce uncertainty and thereby promote economic activity, the policymaker may commit to an exchange rate band or even to a peg. Such commitment, however, is costly since maintenance of the currency within the band occasionally requires the policymaker to use up foreign exchange reserves or deviate from the interest rate level that is consistent with other domestic objectives. The cost of either option rises if the exchange rate comes under a speculative attack. If the policymaker decides to exit the band and avoid the costs of defending it, he losses credibility. The optimal exchange rate regime reflects, therefore, a trade-off between reduction of exchange rate uncertainty and the cost of committing to an exchange rate band or a peg. This trade-off is in the spirit of the escape clause literature (e.g., Lohmann, 1992 and Obstfeld, 1997).

By explicitly recognizing the interdependence between speculative attacks and the choice of exchange rate regime, the framework in the paper yields a number of novel predictions about the optimal exchange rate regime and about the likelihood of a currency attack. For instance, we show that a Tobin tax on short term inter-currency transactions that was proposed by Tobin (1978) as a way to reduce the profitability of speculation against the currency and thereby lower the probability of currency crises, may actually be counterproductive. Although, as suggested by conventional wisdom , the tax *does lower* the likelihood of currency crises for a given band, it also induces policymakers to set narrower bands in order to achieve more ambitious reductions in exchange rate uncertainty.[7] All else equal, this longer run policy response *raises* the probability of crises and, in some reasonable cases, may actually dominate the first effect. In addition the

of countries, including Brazil, Chile, Colombia, Ecuador, Finland, Hungary, Israel, Mexico, Norway, Poland, Russia, Sweden, The Czech Republic, The Slovak Republic, Venezuela and a number of emerging Asian countries.

[6] For the same reason, we also analyze a three-stage model instead of a full-fledged dynamic framework. In utilizing this simplification we follow Obstfeld (1996) and Morris and Shin (1998), who analyze reduced-form two-stage models.

[7] This result is also consistent with the flexibilization of exchange rate regimes following the gradual elimination of restrictions on capital flows in the aftermath of the Bretton Woods system.

paper analyzes the effect of other factors, such as the aversion to exchange rate uncertainty, the variability in fundamentals, the tightness of commitment, and the reputation of policymakers, on the choice of exchange rate regime.

As a byproduct, the paper also contributes to the literature on target zones and exchange rate bands. Most of this literature has focused on the dynamics of exchange rates, interest rates, and central bank interventions within *given* exchange rate bands.[8] By contrast, this paper focuses on the trade-offs that determine the optimal band width. To this end, it abstracts from the effect of a band on the behavior of the exchange rate within the band, which is a main focus of the traditional target zone literature, and focuses instead on the strategic interaction between the ex ante choice of exchange rate regime and the behavior of speculators.[9] We are aware of only three other papers that analyze the optimal width of the band: Sutherland (1995), Miller and Zhang (1996) and Cukierman, Spiegel, and Leiderman (2002). The first two papers do not consider the possibility of realignments, nor the interaction between currency attacks and the optimal width of the band. The third paper incorporates the possibility of realignments, but abstracts from the issue of speculative attacks.

The paper is organized as follows. Section 2 presents the basic framework. Section 3 derives the equilibrium behavior of speculators and of the policymaker and characterizes the equilibrium properties of the exchange rate regime. In particular, this section identifies conditions under which the regime is a peg, a free float, or a band, and in the later case, examines the determinants of the band's width and of its symmetry. Section 4 provides comparative statics analysis and discusses its empirical implications. Section 5 extends the analysis to the case in which speculators are uncertain about the policymaker's resolve to maintain the band (i.e., the policymaker's reputation). Section 6 concludes. All proofs are in the Appendix.

2 The model

Consider an open economy in which the initial level of the nominal exchange rate (defined as the number of units of domestic currency per one unit of foreign currency) is e_{-1}. Absent policy interventions and speculation, the new level of the nominal exchange rate, e, is determined in the

[8] This literature orignated with a seminal paper by Krugman (1991), and continued with many other contributions, such as, Bertola and Caballero (1992), and Bertola and Svensson (1993). See Garber and Svensson (1995) for an extensive literature survey.

[9] From this point of view, this paper and the target zone literature from the early nineties complement each other.

exchange rate markets. The realization of e reflects various shocks to the current account and to the capital account of the balance of payments excluding the behavior of speculators and government interventions, which are modeled explicitly. For the purpose of this paper, it turns out that it is more convenient to work with the laissez faire rate of change in e, $x \equiv \frac{e-e_{-1}}{e_{-1}}$, rather than with its level, e. We assume that x is drawn from a distribution function $f(x)$ on $\Re$ with c.d.f. $F(x)$. We make the following assumption on $f(x)$:

Assumption 1: *$f(x)$ is unimodal with a mode at $x = 0$. That is, $f(x)$ is increasing for all $x < 0$ and decreasing for all $x > 0$.*

Assumption 1 states that large rates of change in the freely floating exchange rate (i.e., large depreciations when $x > 0$ and large appreciations when $x < 0$) are less likely than small changes. This is a realistic assumption and, as we shall see later, it is responsible for some of the results of the paper.

2.1 The exchange rate band

A basic premise of the paper is that policymakers dislike nominal exchange rate uncertainty. This is because exporters, importers, as well as lenders and borrowers in foreign currency face higher exchange rate risks when there is more uncertainty about the nominal exchange rate. By raising the foreign exchange risk premium, an increase in exchange rate uncertainty reduces international flows of goods and of financial capital. Policymakers, who wish to promote economic activity, internalize at least part of this aversion to uncertainty and therefore have an incentive to limit it.[10]

In general, there are various conceivable institutional arrangements for limiting exchange rate uncertainty. In this paper we search for an optimal institutional arrangement within the class of bands. This class is quite broad and includes pegs (bands of zero width) and free floats (bands of infinite width) as special cases. Under this class of arrangements, the policymaker sets an exchange rate band $[\underline{e}, \overline{e}]$ around the preexisting nominal exchange rate, e_{-1}. The nominal exchange rate, e, is then allowed to move freely within the band in accordance with market forces, but if the laissez faire exchange rate is outside the band, the policymaker is committed to intervene and

[10] Admittedly, some of those risks may be insured by means of future currency markets. However, except perhaps for some of the major key currencies, such markets are largely non-existent, and when they do exist the insurance premia are likely to be prohibitively high.

keep the exchange rate at one of the boundaries of the band.[11] Given e_{-1} the exchange rate band induces a permissible range of rates of change in the exchange rate, $[\underline{\pi}, \overline{\pi}]$, where $\underline{\pi} \equiv \frac{\underline{e}-e_{-1}}{e_{-1}} < 0$ and $\overline{\pi} \equiv \frac{\overline{e}-e_{-1}}{e_{-1}} > 0$. Within this range, the domestic currency is allowed to appreciate if $x \in [\underline{\pi}, 0)$, and depreciate if $x \in [0, \overline{\pi})$. In other words, $\underline{\pi}$ is the maximal rate of appreciation and $\overline{\pi}$ is the maximal rate of depreciation that the exchange rate band allows.[12]

But leaning against the trends of free exchange rate markets is costly. To defend a currency under attack, policymakers have to deplete their foreign exchange reserves (Krugman, 1979) or put up with substantially higher domestic interest rates (Obstfeld, 1996). If they decide to avoid those costs by exiting the band, policymakers lose some credibility. This loss makes it harder to achieve other goals either in the same period or in the future (e.g., commit to a low rate of inflation or to low rates of taxation, accomplish structural reforms, etc.). We denote the present value of this loss by δ. Hence δ characterizes the policymaker's aversion to realignments.

After observing the realization of x, and the fraction of speculators who attack the band, the policymaker can either intervene in order to maintain the band or abandon it. Following Obstfeld (1996) and Morris and Shin (1998), we assume that the cost of maintaining the band increases with the size of the disequilibrium that the policymaker tries to maintain (either $x - \overline{\pi}$ or $\underline{\pi} - x$, depending on whether x is positive or negative) and with the number of speculators who attacked the band. Specifically, normalizing the mass of speculators to 1, and using α to denote the fraction of speculators that have attacked the band, we assume that the cost of intervention in the exchange rate market is given by

$$C(x, \alpha) = \begin{cases} x - \overline{\pi} + \alpha, & x \geq \overline{\pi}, \\ 0, & \underline{\pi} \leq x \leq \overline{\pi}, \\ \underline{\pi} - x + \alpha, & x \leq \underline{\pi}. \end{cases} \tag{2.1}$$

The assumption that $C(x, \alpha)$ increases with α reflects the idea that as more speculators attack the band, the policymaker has less resources to continue to defend it, and hence it becomes more costly to maintain it. For simplicity, we assume that α enters the cost function additively. The middle line in equation (2.1) states that when the exchange rate is inside the band, the policymaker does not intervene in the market and bears no cost. Obviously, the policymaker will maintain the band only when $C(x, \alpha) \leq \delta$. Otherwise, the policymaker will exit the band and incur the cost of

[11] This intervention can be operationalized by buying or selling foreign currency in the market, by changing the domestic interest rate, or by doing some of both.

[12] Note that when $\underline{\pi} = \overline{\pi} = 0$ the band reduces to a peg and when $\underline{\pi} = -\infty$, and $\overline{\pi} = \infty$ it becomes a free float.

realignment, δ. Hence, the policymaker's cost of adopting an exchange rate band for a given x is $Min\{C(x,\alpha),\ \delta\}$.

We formalize the trade-off between uncertainty about the nominal exchange rate and the cost of adopting a band by postulating that the policymaker's objective is to select the bounds of the band, $\underline{\pi}$ and $\overline{\pi}$ to maximize

$$V(\underline{\pi},\overline{\pi}) = -AE\left|\pi - E\pi\right| - E\left[Min\{C(x,\alpha),\ \delta\}\right], \qquad A > 0, \tag{2.2}$$

where π is the actual rate of change in the nominal exchange rate (under laissez faire, $\pi = x$). We think of the policymaker's maximization problem mostly as a positive description of how a rational policymaker might approach the problem of choosing the band width. The second component of V is simply the policymaker's expected cost of adopting an exchange rate band. The first component of V represents the policymaker's aversion to nominal exchange rate *uncertainty*, measured in terms of the expected absolute value of unanticipated nominal depreciations/appreciations.[13] The parameter A represents the relative importance that the policymaker assigns to reduction of exchange rate uncertainty and is likely to vary substantially across economies depending on factors like the degree of openness of the economy, its size, the fraction of financial assets and liabilities owned by domestic producers and consumers that are denominated in foreign exchange, and the fraction of foreign trade that is invoiced in foreign exchange (McKinnon, 2000; Gylfason, 2000; and Wagner, 2000). All else equal, residents of small open economies are more averse to nominal exchange rate uncertainty than residents of large, relatively closed, economies like the US or the Euro area. Hence a reasonable presumtion is that A is larger in small open economies than in large, relatively closed, economies.

2.2 Speculators

We model speculative behavior using the Morris and Shin (1998) apparatus. There is a continuum of speculators, each of whom can take a position of at most one unit of foreign currency. The total mass of speculators is normalized to one. When the exchange rate, e, is either at the upper bound

[13] It is important to note that the policymaker is averse to excahnge rate *uncertainty* and not actual exchange rate *variability* (see Cukierman and Wachtel, 1982, for a general distinction between uncertainty and variability). Indeed, this is the reason for commiting to a band ex ante: Without commitment, there is a time inconsistency problem (Kydland and Prescott, 1977, and Barro and Gordon, 1983), so the market will correctly anticipate that, since he is not averse to predictable variability, the policymaker will have no incentive to intervene ex post, after the realization of x.

of the band, $\overline{e}$, or at the lower bound, $\underline{e}$, each speculator independently observes a noisy signal on the exchange rate that would prevail under laissez faire. Specifically, we assume that the signal obtained by speculator i is

$$\theta_i = x + \varepsilon_i, \tag{2.3}$$

where ε_i is a white noise, independent across speculators, and distributed uniformly on the interval $[-\varepsilon, \varepsilon]$. The conditional density of x given a signal θ_i is given by:

$$f(x \mid \theta_i) = \frac{f(x)}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \tag{2.4}$$

In what follows, we focus on the case where ε is small so that the signals that speculators observe are "almost perfect."

Based on θ_i, each speculator i decides whether or not to attack the currency. If e is at $\underline{e}$, speculator i can shortsell the foreign currency at the current (high) price $\underline{e}$ and then buy the foreign currency on the market to clear his position. Denoting by t the nominal transaction cost associated with switching between currencies, the speculator's net payoff is $\underline{e} - e - t$, if the policymaker fails to defend the band and e falls below $\underline{e}$. Otherwise the payoff is $-t$. Likewise, if $e = \overline{e}$, speculator i can buy the foreign currency at the current (low) price $\overline{e}$. Hence, the speculator's net payoff is $e - \overline{e} - t$, if the policymaker exits the band and the exchange rate jumps to $e > \overline{e}$. If the policymaker successfully defends the band his payoff is $-t$. If the speculator does not attack the band, his payoff is 0.[14] We now make the following assumption on t:

Assumption 2: $t < \delta e_{-1}$.

As will become clear below, Assumption 2 ensures that speculators will always attack the band if they believe that x is such that the policymaker will exit the band. This rules out the (uninteresting) possibility that speculators do not attack the band even if they know that the policymaker is not going to defend it.

2.3 The sequence of events and the structure of information

The sequence of events unfolds as follows:

- Stage 1: The policymaker announces a band around the existing nominal exchange rate and commits to intervene when $x < \underline{\pi}$ or $x > \overline{\pi}$.

[14] In order to focus on speculation against the band we abstract from speculative trading within the band. Thus, the well-known 'honeymoon effect' (Krugman, 1991) is absent from the model.

- Stage 2: The "free float" random shock, x, is realized. There are now two possible cases:

  (i) If $\underline{\pi} \leq x \leq \overline{\pi}$, the nominal exchange rate is determined by market forces. Hence, $e = (1 + x)e_{-1}$.

  (ii) If $x < \underline{\pi}$ or $x > \overline{\pi}$, then $e = \underline{e}$, or $e = \overline{e}$, respectively. Simultaneously, each speculator i gets the signal, θ_i, on x and decides whether or not to attack the band.

- Stage 3: The policymaker observes x and the fraction of speculators who decide to attack the band, α, and decides whether or not to defend the band. If he does, e stays at the boundary of the band and the policymaker incurs the cost $C(x, \alpha)$. If the policymaker exits the band, e moves to its freely floating rate so the induced rate of change in the exchange rate is x and the policymaker incurs a future credibility loss whose present value is δ.[15]

3 The equilibrium

To characterize the perfect Bayesian equilibrium of the model, we solve the model backwards. First, whenever $x < \underline{\pi}$ or $x > \overline{\pi}$, then given α, the policymaker decides in stage 3 whether or not to continue to maintain the band. Second, given the signals that they observe in stage 2, speculators decide whether or not to attack the band. When $x \in [\underline{\pi}, \overline{\pi}]$, the policymaker does not intervene in the exchange rate market and the exchange rate moves freely within the band. Finally, in stage 1, prior to the realization of x, the policymaker sets the exchange rate regime.

[15] The events at stages 2 and 3 of the model are similar to those in Morris and Shin (1998), and follow the implied sequence of events in Obstfeld (1996). The assumptions imply that speculators can profit from attacking the currency if there is a realignment, and that the policymaker realigns only if the fraction of speculators who attack is sufficiently large. These realistic features are captured by the model in a reduced-form manner. One possible way to justify these features within our framework is as follows: Initially (at stage 2), the exchange rate policy is on "authomatic pilot," (say due to a short lag in decision making or in the arrival of information) so the policymaker intervenes authomatically as soon as e reaches the boundaries of the band. Speculators buy foreign currency or shortsell it at this point in the hope that a realignment will take place. In stage 3, the policymaker reevaluates his policy by comparing $C(x, \alpha)$ and δ. If $C(x, \alpha) > \delta$, he exits from the band and speculators make a profit on the difference between the price they got in stage 2, and the new price that is set in stage 3. For simplicity, we assume that the cost of intervention in stage 2 is 0. In a previous version we also analyzed the case where the cost of intervention in stage 2 is positive but found that all our results go through.

3.1 Speculative attacks

When $x \in [\underline{\pi}, \overline{\pi}]$, the exchange rate is determined solely by market forces. In contrast, when $x < \underline{\pi}$ or $x > \overline{\pi}$, the policymaker initially intervenes in the foreign exchange market and prevents the exchange rate from moving outside the band. Then the policymaker reevaluates his policy and continues to defend the band if and only if $C(x, \alpha) \leq \delta$. As x reaches either $\underline{\pi}$ or $\overline{\pi}$, speculators realize that the exchange rate no longer reflects market forces and hence may choose to attack the band if they expect that the policymaker will eventually exit the band. But since speculators do not observe x and α directly, each speculator needs to use his signal in order to assess the policymaker's decision on whether to continue to defend the band or abandon it.

Lemma 1 *Suppose that speculators have almost perfect information, i.e., $\varepsilon \to 0$. Then,*

(i) when the exchange rate reaches the upper (lower) bound of the band, there exists a unique perfect Bayesian equilibrium, such that each speculator attacks the band if and only if the signal that he observes is above some threshold $\overline{\theta}^*$ (below some threshold $\underline{\theta}^*$).

(ii) The thresholds $\overline{\theta}^*$ and $\underline{\theta}^*$ are given by $\overline{\theta}^* = \overline{\pi} + r$ and $\underline{\theta}^* = \underline{\pi} - r$, where

$$r = \sqrt{\frac{t}{e_{-1}} + \frac{(\delta - 1)^2}{4}} + \frac{\delta - 1}{2}.$$

(iii) In equilibrium, all speculators attack the upper (lower) bound of the band and the policymaker realigns it if and only if $x > \overline{\theta}^* = \overline{\pi} + r$ $(x < \underline{\theta}^* = \underline{\pi} - r)$. Whenever $\underline{\theta}^* \leq x \leq \overline{\theta}^*$ speculators do not attack the band and the band is not realigned. The probability of a speculative attack is therefore

$$P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r)).$$

The proof of Lemma 1, as well as the proofs of all other results, is in the Appendix. The uniqueness result in part (i), follows from arguments similar to those in Carlsson and van Damme (1993) and Morris and Shin (1998) and is based on an iterative elimination of dominated strategies. The idea is as follows. Suppose that e has reached $\overline{e}$ (the logic when e reaches $\underline{e}$ is analogous). When θ_i is sufficiently large, speculator i correctly anticipates that x is such that the policymaker will surely exit the band even if no speculator attacks it. Hence, it is a dominant strategy for speculator i to attack.[16] But now, if θ_i is slightly lower, speculator i realizes that a large fraction

[16] The existence of a region in which speculators have dominant strategies is crucial for deriving a unique equilibrium (see Chan and Chiu, 2002).

of speculators must have observed even higher signals and will surely attack the band. From that, speculator i concludes that the policymaker will exit the band even at this slightly lower signal, so it is again optimal to attack it. This chain of reasoning proceeds further where each time we lower the critical signal above which speculator i will attack $\overline{e}$. Likewise, when θ_i is sufficiently low, speculator i correctly anticipates that x is so low that the profit from attacking is below the transaction cost t even if the policymaker will surely exit the band. Hence, it is a dominant strategy not to attack at $\overline{e}$. But then, if θ_i is slightly higher, speculator i correctly infers that a large fraction of speculators must have observed even lower signals and will surely not attack at $\overline{e}$. From that, speculator i concludes that the policymaker will successfully defend $\overline{e}$ so again it is optimal not to attack. Once again, this chain of reasoning proceeds further where each time we raise the critical signal below which the speculator will not attack $\overline{e}$.

As $\varepsilon \longrightarrow 0$, the critical signal above which speculators attack $\overline{e}$ coincides with the critical signal below which they do not attack it. This yields a unique threshold signal $\overline{\theta}^*$, such that all speculators attack $\overline{e}$ if and only if they observe signals above $\overline{\theta}^*$. Similar arguments establish the existence of a unique threshold signal, $\underline{\theta}^*$, such that all speculators attack $\underline{e}$ if and only if they observe signals below $\underline{\theta}^*$.

Having characterized the behavior of speculators, we turn next to the implications of this behavior for the exchange rate band. Recall that we are interested in cases where $\varepsilon \to 0$. Part (iii) of Lemma 1 implies that the exchange rate band gives rise to two *Ranges of Effective Commitment* (REC) such that the policymaker intervenes in the exchange rate market and defends the band only if x falls inside one of these ranges. The positive REC is equal to $[\overline{\pi}, \overline{\theta}^*]$ or $[\overline{\pi}, \overline{\pi} + r]$; when $x \in [\overline{\pi}, \overline{\pi}+r]$, the policymaker ensures that the rate of depreciation will not exceed $\overline{\pi}$. The negative REC is equal to $[\underline{\theta}^*, \underline{\pi}]$ or $[\underline{\pi} - r, \underline{\pi}]$; when $x \in [\underline{\pi} - r, \underline{\pi}]$, the policymaker ensures that the rate of appreciation will not exceed the absolute value of $\underline{\pi}$. When $x < \underline{\pi} - r$ or when $x > \overline{\pi} + r$, the policymaker exits the band and despite his earlier announcement, tolerates a realignment. On the other hand, when $x \in [\underline{\pi}, \overline{\pi}]$, the policymaker allows the exchange rate to move freely in accordance with market forces. These five ranges of x are illustrated in Figure 1.

Part (ii) of Lemma 1 indicates that r is independent of $\underline{\pi}$ and $\overline{\pi}$. This means that the actual size of the two RECs does not depend on how wide the band is. But, by choosing $\underline{\pi}$ and $\overline{\pi}$ appropriately, the policymaker can shift the two RECs either closer to or away from 0. Part (ii) of Lemma 1 also shows that r increases with t and with δ. These properties are intuitive since they imply that a realignment is less likely when it is more costly for speculators to attack the band and when the policymaker is more averse to realignments.

The discussion is now summarized in the following proposition:

Proposition 1 *The exchange rate band gives rise to a positive range of effective commitment (REC),* $[\overline{\pi}, \overline{\pi}+r]$*, and a negative REC,* $[\underline{\pi}-r, \underline{\pi}]$*, where* r *is defined in Lemma 1.*

- *When* x *falls inside the positive (negative) REC, the policymaker defends the currency and ensures that the maximal rate of depreciation (appreciation) is* $\overline{\pi}$ *(*$\underline{\pi}$*).*
- *When* x *falls below the negative REC, above the positive REC, or inside the band, the policymaker lets the exchange rate move freely in accordance with market forces.*
- *The width of the two RECs,* r*, increases with* t *and with* δ *but is independent of the boundaries of the band,* $\underline{\pi}$ *and* $\overline{\pi}$*.*

3.2 The choice of band width

In order to characterize the equilibrium exchange rate regime, we first need to write the policymaker's objective function, $V(\underline{\pi}, \overline{\pi})$, more explicitly. The first component in $V(\underline{\pi}, \overline{\pi})$ represents the policymaker's aversion to exchange rate uncertainty. This term depends on the expected rate of change in the exchange rate, $E\pi$, which in turn depends on the policymaker's choices, $\underline{\pi}$ and $\overline{\pi}$. At first blush one may think that, since $\underline{\pi}$ is the maximal rate of appreciation and $\overline{\pi}$ is the maximal rate of depreciation, $E\pi$ will necessarily lie between $\underline{\pi}$ and $\overline{\pi}$. However, since the policymaker does not always defend the band, $E\pi$ may in principle fall outside the interval $[\underline{\pi}, \overline{\pi}]$. Consequently in writing $V(\underline{\pi}, \overline{\pi})$ we need to distinguish between 5 possible cases depending on whether $E\pi$ falls inside the interval $[\underline{\pi}, \overline{\pi}]$, inside one of the two RECs, below the negative REC, or above the positive REC. Fortunately, the following lemma simplifies the analysis considerably.

Lemma 2 *If the optimal exchange rate regime is* not *a free float then* $E\pi \in [\underline{\pi}, \overline{\pi}]$.

Given Lemma 2, the measure of exchange rate uncertainty in any regime that is not a free float is given by:

$$\begin{aligned} E\left|\pi - E\pi\right| \;=\; & -\int_{-\infty}^{\underline{\pi}-r} (x - E\pi)\, dF(x) - \int_{\underline{\pi}-r}^{\underline{\pi}} (\underline{\pi} - E\pi)\, dF(x) - \int_{\underline{\pi}}^{E\pi} (x - E\pi)\, dF(x) \\ & + \int_{E\pi}^{\overline{\pi}} (x - E\pi)\, dF(x) + \int_{\overline{\pi}}^{\overline{\pi}+r} (\overline{\pi} - E\pi)\, dF(x) + \int_{\overline{\pi}+r}^{\infty} (x - E\pi)\, dF(x). \end{aligned} \tag{3.1}$$

Equation (3.1) and (A13) in the Appendix show that the existence of a band affects uncertainty only through its effect on the two RECs.

Using (2.2) and (3.1), the expected payoff of the policymaker, given $\underline{\pi}$ and $\overline{\pi}$, becomes

$$\begin{aligned} V(\underline{\pi},\overline{\pi}) &= A\Bigg[\int_{-\infty}^{\underline{\pi}-r}(x-E\pi)\,dF(x)+\int_{\underline{\pi}-r}^{\underline{\pi}}(\underline{\pi}-E\pi)\,dF(x)+\int_{\underline{\pi}}^{E\pi}(x-E\pi)\,dF(x) \\ &\quad -\int_{E\pi}^{\overline{\pi}}(x-E\pi)\,dF(x)-\int_{\overline{\pi}}^{\overline{\pi}+r}(\overline{\pi}-E\pi)\,dF(x)-\int_{\overline{\pi}+r}^{\infty}(x-E\pi)\,dF(x)\Bigg] \qquad (3.2) \\ &\quad -\int_{-\infty}^{\underline{\pi}-r}\delta dF(x)-\int_{\underline{\pi}-r}^{\underline{\pi}}(\underline{\pi}-x)dF(x)-\int_{\overline{\pi}}^{\overline{\pi}+r}(x-\overline{\pi})dF(x)-\int_{\overline{\pi}+r}^{\infty}\delta dF(x), \end{aligned}$$

where the last line represents the expected cost of adopting a band: when x falls inside the two RECs, the policymaker incurs a cost of intervention in the exchange rate market, which is $\underline{\pi}-x$ when $x\in[\underline{\pi}-r,\underline{\pi}]$ or $x-\overline{\pi}$ when $x\in[\overline{\pi},\overline{\pi}+r]$. When either $x<\underline{\pi}-r$ or $x>\overline{\pi}+r$, there are realignments so the policymaker incurs a credibility loss, δ.

The policymaker chooses the boundaries of the band, $\underline{\pi}$ and $\overline{\pi}$, so as to maximize V. The first order conditions for an interior solution (i.e., for $-\infty<\underline{\pi}<0<\overline{\pi}<\infty$) are:

$$\begin{aligned} \frac{\partial V(\underline{\pi},\overline{\pi})}{\partial\underline{\pi}} &= -2A(F(E\pi)-1)\int_{\underline{\pi}-r}^{\underline{\pi}}(f(x)-f(\underline{\pi}-r))\,dx \qquad (3.3) \\ &\quad -\int_{\underline{\pi}-r}^{\underline{\pi}}(f(x)-f(\underline{\pi}-r))\,dx-\delta f(\underline{\pi}-r)=0, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial V(\underline{\pi},\overline{\pi})}{\partial\overline{\pi}} &= -2AF(E\pi)\int_{\overline{\pi}}^{\overline{\pi}+r}(f(x)-f(\overline{\pi}+r))\,dx \qquad (3.4) \\ &\quad +\int_{\overline{\pi}}^{\overline{\pi}+r}(f(x)-f(\overline{\pi}+r))\,dx+\delta f(\overline{\pi}+r)=0. \end{aligned}$$

In Lemma A1 of the Appendix we show that $f"(x)\le 0$ for all x and $A>Max\left[\frac{1}{2F(E\pi)},\frac{1}{2(1-F(E\pi))}\right]$, along with Assumption 1, are sufficient conditions for $V(\underline{\pi},\overline{\pi})$ to be globally concave, in which case (3.3) and (3.4) are sufficient for a unique maximum. Equations (3.3) and (3.4) show that by altering the bounds of the band, the policymaker trades-off the benefits of reducing exchange rate uncertainty against the cost of maintaining a band. The term in the first line of (3.4) is the marginal effect of $\overline{\pi}$ on exchange rate uncertainty. Since by Assumption 1, $f(x)-f(\overline{\pi}+r)>0$ for all $x\in[\overline{\pi},\overline{\pi}+r]$, this term is negative and represents the marginal cost from raising $\overline{\pi}$. This marginal cost arises because when $\overline{\pi}$ is raised, the positive REC over which the exchange rate is kept constant, shifts further away from the center rate to a range of shocks that is less likely by Assumption 1. Therefore, the shift of the positive REC away from 0 makes the band less effective in reducing exchange rate uncertainty. The second line in (3.4) represents the marginal effect of raising $\overline{\pi}$ on the cost of defending the band. By Assumption 1, the integral term in the brackets is positive, implying that raising $\overline{\pi}$ makes it less costly to defend the band. This is because now it is less likely that the policymaker will actually have to defend the band. The term involving δ is also positive since increasing $\overline{\pi}$ slightly lowers the likelihood that the exchange rate will move outside the positive REC and lead to a realignment. The interpretation of (3.3) is analogous except that here, the signs of the various terms are exactly opposite since raising $\underline{\pi}$ slightly shifts the lower bound of the band closer to 0, whereas raising $\overline{\pi}$ slightly shifts the upper bound of the band away from 0.

Using (3.3) and (3.4), we establish the following result:

Lemma 3 *If $f(x)$ is symmetric around 0, the equilibrium exchange rate band will be symmetric around 0 in the sense that $-\underline{\pi} = \overline{\pi}$.*[17] *Consequently, $E\pi = 0$.*

To simplify the exposition, we shall restrict attention from now on to the following case:[18]

Assumption 3: *The distribution $f(x)$ is symmetric around 0.*

Together with Assumption 1 that states that x is unimodal with a mode at 0, Assumption 3 implies that the mean of x is 0 (on average, there is no pressure for either appreciations nor depreciations). Moreover, since both $f(x)$ and the band are symmetric, $E\pi = 0$, so in expectations, appreciations and depreciations are equally likely. Given Assumption 3, we now characterize the equilibrium exchange rate regime.

Proposition 2 *The equilibrium exchange rate band has the following properties:*

(i) ***Free float:*** *If $A \leq 1$, then $\underline{\pi} = -\infty$ and $\overline{\pi} = \infty$, so the optimal regime is a free float.*

(ii) ***A nondegenerate band:*** *If*

$$1 < A < \overline{A}(r) \equiv 1 + \frac{\delta}{\int_0^r \left[\frac{f(x)}{f(r)} - 1\right] dx}, \tag{3.5}$$

then $-\infty < \underline{\pi} < 0 < \overline{\pi} < \infty$. Hence, the optimal regime is a nondegenerate band.

[17]Note that the symmetry of the band is formulated in terms of the permissible rates of appreciations and depreciations of the exchange rate rather than in terms of the gap of the upper and lower bounds of the band from the center rate (i.e., the symmetry is formulated in terms of x rather than in terms of e).

[18]It should be noted that the general qualitative nature of our results extends to the case where $f(x)$ is asymmetric. But the various mathematical expressions and conditions become more complex.

(iii) ***A peg:*** *If $f''(x) \leq 0$ for all x and $A > \overline{A}(r)$, then $\underline{\pi} = \overline{\pi} = 0$, so the optimal regime is a peg.*

Part (i) of Proposition 2 states that when the policymaker has sufficiently little concern for nominal exchange rate uncertainty, i.e., $A \leq 1$, he sets a free float and completely avoids the cost of maintaining a band. Part (ii) of the proposition states that for intermediate values of A, the policymaker balances the benefits from reducing exchange rate uncertainty against the cost of intervention in the exchange rate market by setting a nondegenerate band. Intervention occurs only when x falls inside the negative or the positive RECs. Part (iii) of Proposition 2 states that if the policymaker is highly concerned with nominal exchange rate uncertainty, i.e., $A > \overline{A}(r)$, his best strategy is to adopt a peg.[19]

4 Comparative statics and empirical implications

4.1 The effects of restrictions on capital flows and of a Tobin tax

During the last three decades there has been a world-wide gradual lifting of restrictions on currency flows and on related capital account transactions. One consequence of this trend is a reduction in the transaction cost of foreign exchange transactions (t in terms of the model) making it easier for speculators to move funds across different currencies, thereby increasing the likelihood of speculative attacks. To counteract this tendency some economists proposed to "throw sand" into the wheels of unrestricted international capital flows. In particular, Tobin (1978) proposed a universal tax on short term inter-currency transactions in order to reduce the profitability of speculation against the currency, and with it the probability of crises. This idea was met with scepticism mainly because of difficulties of implementation. But, by and large, the consensus is that, subject to feasibility, the tax can reduce the probability of attack on the currency. Recent evaluations appear in Eichengreen, Tobin, and Wyplosz (1995), Jeanne (1996), Haq, Kaul and Grunberg (1996), Eichengreen (1999) and Berglund et al. (2001).

The main objective of this subsection is to examine the consequences of such a tax and of the lifting of restrictions on capital flows when the choice of exchange rate regime is endogenous. More formally, the following proposition examines the effect of a reduction in t on the choice of

[19] Note that a peg does not mean that the exchange rate is fixed under all circumstances. When the absolute value of x exceeds r, the policymaker abandons the peg and the exchange rate is realigned. Hence, under a peg, the exchange rate is fixed for all $x \in [-r, r]$. Given Assumption 1, such "small" shocks are more likely than big ones, so when A is large, it is optimal for the policymaker to eliminate these shocks by adopting a peg.

exchange rate regime and on the likelihood of currency crises.

Proposition 3 *Suppose that following a lifting of restrictions on currency flows and capital account transactions, the transaction cost of switching between currencies, t, decreases. Then:*

(i) When the policymaker's problem has a unique interior solution, $\overline{\pi}$ and $\underline{\pi}$ shift away from 0, so the band becomes wider. Moreover, the probability, P, that a speculative attack occurs decreases.

(ii) The bound $\overline{A}(r)$ above which the policymaker adopts a peg increase, implying that policymakers adopts pegs for a narrower range of values of A.

(iii) The equilibrium value of the policymaker's objectives, V, falls.

Part (i) of Proposition 3 states that, in our framework, lifting restrictions on the free flow of capital lowers, on balance, the likelihood of a currency crisis. This counterintuitive result is the outcome of two opposing effects. First, as Proposition 1 shows, the two RECs shrink when t decreases. Holding the band width constant, this raises the probability of speculative attacks. This effect already appears in the recent literature on international financial crises (e.g., Morris and Shin, 1998). But since the two RECs shrink, the band becomes less effective in guaranteeing exchange-rate stability. The policymaker's reaction to this is to pursues less ambitious stabilization objectives and allow the exchange rate to move freely within a wider band. This lowers, in turn, the probability, P, of speculative attacks. Thus, in general, the effect of reducing t on the probability of a currency attack is ambiguous. Part (i) of Proposition 3 suggests that, for non degenerate bands and symmetric distributions of fundamentals, the second effect dominates so P decreases when t is reduced.[20] When $f(x)$ is not necessarily symmetric the same two opposing effects on the probability of speculative attack are still operative but the sign of their combined effect on the probability of attack is generally ambiguous.

This result implies that, when the endogeneity of the exchange rate regime is recognized, conventional wisdom may be reversed. Dismantling of restrictions on capital flows may reduce rather than raise the probability of currency crisis. The mirror image of this result is that the imposition of a Tobin tax generally has an ambiguous effect on the likelihood of crisis. For a given

[20] This result is partly reminiscent of the discussion in Kupiec (1996) which establishes that, when general equilibrium effects are taken into consideration, a securities transaction tax does not necessarily reduce stock return volatility.

band width, the tax **does reduce** the likelihood of a currency crisis. However, the imposition of the tax also induces policymakers to install narrower bands in order to achieve more ambitious reductions in exchange rate uncertainty. All else equal, this longer run policy response **raises** the probability of crises at least for symmetric distributions of fundamentals.

Part (ii) of Proposition 3 predicts that, for symmetric distributions of fundamentals, liberalization of the capital account, as characterized by a reduction in t, should lead to a narrowing of the set of countries that maintain pegs. It also implies that, in spite of this trend, countries with strong preference for stability of the exchange rate (e.g., small open economies with relatively large shares of foreign currency denominated trade and capital flows as well as emerging markets) will continue to peg even in the face of capital market liberalization. By contrast, countries with intermediate preferences for exchange rate stability (e.g., more financially mature economies with a larger fraction of domestically denominated debt and capital flows) will move from pegs to bands. These predictions seem to be consistent with casual evidence. Two years following the 1997/8 East Asian crisis, most emerging markets countries in that region are back on pegs (McKinnon, 2001 and Calvo and Reinhart, 2002). On the other hand, following the EMS currency crisis at the beginning of the 90's, the system of cooperative pegs that had existed prior to the crisis was replaced by wide bands until the formation of the EMU at the beginning of 1999.

Finally, part (iii) of Proposition 3 shows that although a decrease in t lowers the likelihood of a financial crisis, it nonetheless can make the policymaker worse-off. The reason is that speculative attacks impose a constraint on the policymaker when he chooses the optimal exchange rate regime. A decrease in t strengthens the incentive to mount a speculative attack and thereby makes this constraint more binding.

4.2 The effects of aversion to exchange rate uncertainty

In this subsection we turn to the effects of the parameter A (the relative importance that the policymaker assigns to reduction of exchange rate uncertainty) on the choice of regime. As we argued earlier, residents of small open economies with large fractions of assets and liabilities denominated in foreign exchange, are more averse to nominal exchange rate uncertainty than residents of large, relatively closed, economies whose financial assets and liabilities are more likely to be denominated in domestic currency. Hence the parameter A reflects the size of the economy and the degree to which it is open with larger values of A being associated with smaller and more open economies. In what follows we restrict the comparative statics analysis to the case in which the policymaker's problem has a unique interior solution, i.e., $-\infty < \underline{\pi} < 0 < \overline{\pi} < \infty$. As Proposition 3 indicates, this requires A to be larger than 1 but not by "too much."

Proposition 4 *Suppose that the policymaker's problem has a unique interior solution. Then as A increases (the policymaker becomes more concerned with exchange rate stability):*

(i) $\overline{\pi}$ and $\underline{\pi}$ shift closer to 0 so the band becomes tighter, and

(ii) the probability, P, that a speculative attack will occur increases.

Proposition 4 says that as the policymaker becomes more concerned with reduction of uncertainty, he sets a tighter band and allows the exchange rate to move freely within a narrower range around the center rate.[21] Part (ii) of the proposition shows that this tightening of the band raises the likelihood of a speculative attack. This implies that *all else equal*, policymakers in countries with larger values of A are willing to set tighter bands and face a higher likelihood of speculative attacks than policymakers in otherwise similar countries with lower values of A.

Note that as Proposition 2 shows, when A increases above $\overline{A}(r)$, the optimal band width becomes 0 so the optimal regime is a peg. On the other hand, when A falls below 1, the optimal band width becomes infinite so the optimal regime is a free float. Given that a substantial part of international trade is invoiced in US Dollars (McKinnon, 1979), it is likely that policymakers of a key currency country like the US are going to be less sensitive to nominal exchange rate uncertainty and therefore have a smaller A than policymakers in small open economies. Therefore, our model predicts that the US, Japan, and the Euro area should be floating, while Hong-Kong, Panama, Estonia, Lithuania, and Bulgaria, should be on either pegs, currency boards, or even full dollarization. This prediction appears to be consistent with casual observation on the exchange rate systems chosen by those countries..

4.3 The effects of increased variability in fundamentals

Next, we examine how the exchange rate band changes when more extreme realizations of x become more likely. This comparative statics exercise involves shifting probability mass from moderate realizations of x that do not lead to realignments to more extreme realizations that do lead to realignments.

[21] This result may appear obvious at first blush. But the fact that it obtains only under the assumption of unimodality (Assumption 1) suggests that such preliminary intuition is incomplete in the absence of suitable restrictions on the distribution of fundamentals.

Proposition 5 *Suppose that $f(x)$ and $g(x)$ are two symmetric density functions with a mode (and a mean) at 0. Moreover, suppose that $g(x)$ lies above $f(x)$ for all $x < \underline{\pi} - r$ and all $x > \bar{\pi} + r$, where $\underline{\pi}$ and $\bar{\pi}$ are the solutions to the policymaker's problem under the original density function $f(x)$ (that is, $g(x)$ has fatter tails than $f(x)$). Then, the policymaker adopts a wider band under $g(x)$ than under $f(x)$.*

Intuitively, when more extreme realizations of x become more likely (the density of x is $g(x)$ rather than $f(x)$), the policymaker is more likely to incur the loss of future credibility associated with realignments. Therefore, the policymaker widens the band to lower the probability that a costly realignment will take place. Moreover, as larger shocks become more likely, the policymaker finds it optimal to shift the two REC's away from 0 in order to shift his commitment to intervene in the market to a range of shocks that are now more probable. This move benefits the policymaker by counteracting part of the increased uncertainty about the free float value of the exchange rate.

4.4 The effects of tightness of commitment to maintain the regime

The degree of commitment to the exchange rate regime is represented in our model by the parameter δ. Assuming that $f(x)$ is symmetric (in which case $E\pi = 0$) and totally differentiating equations (3.3) and (3.4) with respect to δ reveals that in general, δ has an ambiguous effect on the optimal width of the band. On one hand, as δ increases, speculators attack the band for a smaller range of x. This effect increases the policymaker's incentive to adopt a narrow band. On the other hand, as δ increases, the cost of realignments (when they occur) increases, since they lead to a larger future credibility loss. This effect pushes the policymaker to widen the band. Overall then, the width of the band may either increase or decrease with δ.

Since the probability of speculative attacks, P, is affected by the width of the band, the effect of δ on P is also ambiguous. For a given regime, Proposition 1 implies that P decreases with δ. However, when the endogeneity of the regime is recognized, there is an additional effect that may reverse this result: When δ increases, the policymaker may decide to set a narrower band, knowing that, given the width of the band, he will subsequently decide to maintain the regime for a larger set of values of x. This, in turn, may increase the ex ante probability of a speculative attack. Consequently, an increase in the tightness of commitment may increase the probability of speculative attack.[22]

[22] We also tried to use the model in order to characterize the optimal degree of commitment to the regime but,

5 The exchange rate band and the policymaker's reputation

Typically the public is not fully informed about the commitment ability of policymakers. In this section, we examine how this uncertainty affects the optimal exchange rate regime. To this end, we assume that the policymaker can either be opportunistic in which case $\delta = 0$ or can be dependable in which case $\delta > 0$. Clearly, an opportunistic policymaker will never defend the band when x falls outside the band.[23] We assume that speculators assign a probability β to the policymaker's type being dependable and, following Barro (1986), interpret β as a measure of the policymaker's "reputation."

We now examine how the optimal exchange rate regime is affected by changes in β. As a point of reference, it should be noted that the analysis in previous sections refered to the case where $\beta = 1$. Before proceeding further, we modify Assumption 2 as follows:

Assumption 4: *The real transaction cost, t, is such that* $\delta(1-\beta)e_{-1} < t < \delta e_{-1}$.

Assumption 4 ensures that speculators will always attack the band if they believe that x is such that a dependable policymaker will exit the band, but never attack it if they believe that x is such that the dependable policymaker will defend the band.

5.1 The RECs in the presence of imperfect reputation

With Assumption 4 in place, we examine how the presence of an opportunistic policymaker affects the decisions of speculators on when to attack the band.

Lemma 4 *Suppose that* $\varepsilon \to 0$. *Then,*

(i) speculators will attack the upper bound of the band if and only if they observe signals above $\overline{\theta}^*_\beta$. *They will attack the lower bound of the band if and only if they observe signals below* $\underline{\theta}^*_\beta$.

since the ratio of economic insights to algebra was low this experiment is not presented. Cukierman, Kiguel and Liviatan (1992) and Flood and Marion (1999) present such an analysis for exogenously given pegs. The analysis here is more complex due to the fact that it involves the simultaneous choice of band width as well as of the degree of commitment to the band.

[23] Note however that even an opportunistic policymaker who does not intend to defend the band, will put the initial defense in stage 2 on "automatic pilot" in order to prevent the public from separating him at the outset from his dependable counterpart.

The two thresholds, $\overline{\theta}^*_\beta$ and $\underline{\theta}^*_\beta$, are given by $\overline{\theta}^*_\beta = \overline{\pi} + r^\beta$, and $\underline{\theta}^*_\beta = \underline{\pi} - r^\beta$, where

$$r^\beta = \sqrt{\frac{t}{\beta e_{-1}} + \frac{(\delta - \frac{1}{\beta})^2}{4}} + \frac{\delta - \frac{1}{\beta}}{2}. \tag{4.1}$$

(ii) r^β increases with β.

The behavior of speculators implies that the positive REC is now given by $[\overline{\pi}, \overline{\pi} + r^\beta]$ while the negative REC becomes $[\underline{\pi} - r^\beta, \underline{\pi}]$. Part (ii) of the proposition indicates that the two RECs become narrower when the policymaker's reputation deteriorates.[24] Intuitively, this is because speculators believe that the policymaker will exit the band with a higher probability, so the expected gain from attacking the band is now larger. Hence, the worse is a policymaker's reputation, the more difficult it is to defend the band.

Part (ii) of Lemma 4 can be used to explain how abrupt changes in the policymaker's reputation can generate currency crises even if there is no change in the fundamental, x. To illustrate, consider the 1994 Mexican Peso crisis. Prior to the crisis, Mexico maintained a peg for several years and therefore developed a fair amount of reputation about its resolve to defend it. This together with, inter alias, substantial interest rate differentials attracted a large capital inflow into Mexico. When the ruling party's presidential candidate, Luis Donaldo Colosio, was assassinated in March 1994, the Mexican Peso came under attack. The authorities defended the Peso initially, but, following a substantial loss of reserves within a short period of time, allowed the Peso to float.[25] Second-generation models of currency crises might interpret Colosio's assassination as a sunspot that, for some unexplained reason, was used by speculators as a coordinating device to move the Mexican economy from a good equilibrium to a bad one.

Our framework provides an alternative explanation: Prior to Colosio's assassination, fundamentals were already stretched so that in the absence of intervention the Peso would have depreciated. But, since reputation was high, speculators anticipated that the Mexican government would defend the peg and therefore refrained from attacking it. However, the assassination and the subsequent political instability led to an abrupt decrease in reputation, β. Part (ii) of Lemma 4 suggests that this might have narrowed the REC around the Mexican peg and therefore created

[24] This is analogous to a result in Cukierman and Liviatan (1991) in the context of a Barro-Gordon (1983) inflation bias equilibrium in which the public is uncertain about the dependability of policymakers. Cukierman and Liviatan show that the lower the reputation of a (dependable) policymaker the less ambitious is his inflation target.

[25] A factually based analysis of the sequence of political and economic events preceding the Mexican crisis appears in Whitt (1996).

a new situation in which the free market rate, x, fell outside the REC. Consequently, it became rational for speculators to run on the Peso, and for the Mexican government not to defend it.[26] Note that this interpretation does not rely on sunspots and multiple equilibria and holds even if the assassination had no effect on the economic fundamentals in Mexico.

5.2 Choice of exchange rate regime in the presence of imperfect reputation

When the policymaker's reputation is imperfect exchange rate uncertainty is due to both uncertainty about the shock, x, as well as to uncertainty about the dependability of policymakers. The second source of uncertainty is reflected through the fact that with probability β the policymaker is dependable and will defend the band against speculative attacks whenever x falls inside the two RECs, and with probability $1-\beta$ the policymaker is opportunistic and will never defend the band. Hence, for a symmetric distribution of x, the measure of exchange rate uncertainty is:

$$\begin{aligned} E^{\beta}\left|\pi\right| &= -\int_{-\infty}^{\underline{\pi}-r^{\beta}} xdF(x) - \int_{\underline{\pi}-r^{\beta}}^{\underline{\pi}} \left[\beta\underline{\pi}+(1-\beta)x\right]dF(x) - \int_{\underline{\pi}}^{0} xdF(x) \\ &\quad + \int_{0}^{\overline{\pi}} xdF(x) + \int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} \left[\beta\overline{\pi}+(1-\beta)x\right]dF(x) + \int_{\overline{\pi}+r^{\beta}}^{\infty} xdF(x). \end{aligned} \tag{4.2}$$

Note that β affects $E^{\beta}\left|\pi\right|$ both through its effect on the width of the two RECs and through its effect on the expected change in the exchange rate inside the two RECs.

Given $E^{\beta}\left|\pi\right|$, the expected payoff of a dependable policymaker becomes,

$$\begin{aligned} V^{\beta}(\underline{\pi},\overline{\pi}) &= A\left[\int_{-\infty}^{\underline{\pi}-r^{\beta}} xdF(x) + \int_{\underline{\pi}-r^{\beta}}^{\underline{\pi}} \left[\beta\underline{\pi}+(1-\beta)x\right]dF(x) + \int_{\underline{\pi}}^{0} xdF(x) \right. \\ &\quad \left. - \int_{0}^{\overline{\pi}} xdF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} \left[\beta\overline{\pi}+(1-\beta)x\right]dF(x) - \int_{\overline{\pi}+r^{\beta}}^{\infty} xdF(x)\right] \\ &\quad - \int_{-\infty}^{\underline{\pi}-r^{\beta}} \delta dF(x) - \int_{\underline{\pi}-r^{\beta}}^{\underline{\pi}} (\underline{\pi}-x)dF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} (x-\overline{\pi})dF(x) - \int_{\overline{\pi}+r^{\beta}}^{\infty} \delta dF(x). \end{aligned} \tag{4.3}$$

A dependable policymaker chooses the boundaries of the band, $\underline{\pi}$ and $\overline{\pi}$, so as to maximize $V(\underline{\pi},\overline{\pi})$. We do not need to specify the expected payoff of an opportunistic policymaker because, given that he does not intend to defend the band, he always wishes to announce the same band as his dependable

[26] Another example of a crisis triggered by political events that led to an abrupt change in reputation is the rejection of the Maastricht Treaty by Danish voters in summer of 1992 and the evaluation, at that time, that French voters might also reject the Treaty at the subsequent referendum in September 1992. Isard (1995, p. 210) notes that: "Prudent investors who had earlier sought higher yields by placing funds in potentially vulnerable currencies increasingly saw the merit of covering their exposed positions before the French vote."

counterpart in order to prevent his type from being revealed. This lowers, in turn, exchange rate uncertainty relative to the case where the opportunistic policymaker's type is revealed.

Proposition 6 *The equilibrium exchange rate band has the following properties:*

(i) ***Free float:*** *If* $A\beta \leq 1$*, then* $\underline{\pi} = -\infty$ *and* $\overline{\pi} = \infty$*, so the optimal regime is a free float.*

(ii) ***A nondegenerate band:*** *If*

$$1 < A\beta < 1 + \frac{\delta}{\int_{-r^{\beta}}^{0} \left[\frac{f(x)}{f(-r^{\beta})} - 1\right] dx} = 1 + \frac{\delta}{\int_{0}^{r^{\beta}} \left[1 - \frac{f(x)}{f(r^{\beta})}\right] dx} \equiv A^{\beta},$$

then $0 < -\underline{\pi} = \overline{\pi} < \infty$*. Hence, the optimal regime is a nondegenerate band.*

(iii) ***A peg:*** *If* V^{β} *is concave in* $\underline{\pi}$ *and in* $\overline{\pi}$ *and* $A\beta > A^{\beta}$ *then* $\underline{\pi} = \overline{\pi} = 0$*, so the optimal regime is a peg.*

(iv) ***The width of the band and the likelihood of speculative attacks:*** *Suppose that the policymaker's problem has a unique interior solution. Then,* $\overline{\pi}$ *and* $\underline{\pi}$ *(uniformly) shift closer to* 0 *as* β *increases towards* 1*, implying that as the policymaker's reputation improves, he adopts a tighter band. Moreover, as the policymaker's reputation improves, the likelihood of a speculative attack increases (*$\frac{\partial P^{\beta}}{\partial \beta} > 0$*).*

Parts (i)-(iii) of Proposition 6 modify the corresponding parts of Proposition 2 for the case where the policymaker's reputation is imperfect. Part (iv) of Proposition 6 says that as the policymaker's reputation improves, the exchange rate band becomes tighter. This implies that a good reputation induces the policymaker to be more ambitious in his attempt to reduce exchange rate uncertainty. The reason for that is twofold. First, when the policymaker's reputation improves, a tighter band has a greater moderating effect on the expected variability of the nominal exchange rate. Second, holding the width of the band constant, improved reputation lowers the likelihood of speculative attacks, and therefore makes it less costly for the policymaker to set a tighter band. Hong-Kong's currency board fits into this "box" of the model. Since it has never abandoned its currency board in the past, Hong-Kong's currency board has good reputation, which induces the authorities to defend the currency under a wider set of circumstances than is the case under a lower reputation level. There is thus a "virtuous circle" between good reputation and the performance of a currency board.

But part (iv) of Proposition 6 also states that when a policymaker has a better reputation there is an overall increase in the likelihood of speculative attacks. Although better reputation leads to wider RECs (i.e., ranges of x for which the policymaker defends the band), it also induces the policymaker to adopt tighter bands, which makes the exchange rate regime more susceptible to speculative attacks. For symmetric distributions of fundamentals, the second effect is stronger, so overall there is an increase in the likelihood of speculative attack.

6 Concluding reflections

This paper develops a framework for analyzing the interaction between the *ex ante* choice of exchange rate regime and the probability of *ex post* currency attacks. To the best of our knowledge, this is the first paper that solves endogenously for the optimal regime and for the probability of currency attacks and studies their interrelation.

Our framework generates several novel predictions that are consistent with empirical evidence. First, we find that financial liberalization that lowers the transaction costs of switching between currencies induces the policymaker to adopt a more flexible exchange rate regime. This is broadly consistent with the flexibilization of exchange rate regimes following the gradual reductions of restrictions on capital flows in the aftermath of the Bretton Woods system (see, for example, Isard, 1995). Second, in our model, small open economies with substantial aversion to exchange rate uncertainty are predicted to have narrower bands and more frequent currency attacks than large, relatively closed economies. This is broadly consistent with the fact that large economies with key currencies like the US, Japan and the Euro area chose to float, while small open economies like Argentina (until the beginning of 2002), Thailand, and Korea chose less flexible regimes that are more susceptible to currency attacks like the 1997/8 South-East Asian crisis. Third, the model predicts that policymakers with high reputation tend to set less flexible regimes, and are less vulnerable to speculative attacks. Hong-Kong's currency board is a good example. Since it has never abandoned its currency board in the past, Hong-Kong's currency board enjoys a good reputation, and attracts less speculative pressure.

Another possible prediction of our model, which we did not highlight so far, is related to the bipolar view. According to this hypothesis, following the process of globalization, there has been a gradual shift away from intermediate exchange rate regimes to either hard pegs or freely floating regimes (Fischer, 2001). Globalization is expected to have two opposite effects in our model: On one hand, it lowers the cost of switching between currencies and hence facilitates speculation. This effect induces policymakers to set more flexible regimes. On the other hand, globalization also increases the volume of international trade in goods and financial assets, and increases, therefore, the aversion to nominal exchange rate uncertainty. This effect induces policymakers to set less flexible regimes. The second effect is likely to be large for small open economies whose currencies are not used much for either capital account or current account transaction in world markets, and to be small or even negligible for large key currency economies. Hence, the first effect is likely to be dominant in large, relatively closed blocks, while the second is likely to be dominant in small open economies. All else equal, the process of globalization should therefore induce relatively large currency blocks to move towards more flexible exchange rate arrangements while pushing small open economies in the opposite direction.

The model also implies that, contrary to conventional wisdom, a Tobin tax may actually raise the probability of currency attacks. Although, as in existing literature, a Tobin tax reduces the probability of a currency attack for a given exchange rate regime, the analysis also implies that the tax induces policymakers to set less flexible regimes. Hence, once the choice of an exchange rate regime is endogenized, the tax has an additional, adverse effect on the likelihood of a currency attack which may dominate the first beneficial effect. Similarly, conventional wisdom suggests that when policymakers bear a larger credibility loss following a realignment, they have a stronger incentive to defend the exchange rate regime against speculative attacks and this lowers the probability of this event. However, once the choice of a regime is endogenized, the overall effect becomes ambiguous since ex ante, realizing that speculative attacks are less likely, policymakers may have an incentive to adopt a less flexible regime.

Although our framework captures many empirical regularities regarding exchange rate regimes and speculative attacks, it obviously does not capture all of them. For example, as Calvo and Reinhart (2002) have recently shown, policymakers often intervene in exchange rate markets even in the absence of explicit pegs or bands. We believe that an extension that will analyze the desirability of implicit bands as well as other regimes is a promising direction for future research.[27] Another interesting direction for future research is the development of a dynamic framework in which the fundamentals of the economy are changing over time, and speculators can attack the currency at several points in time. The optimal policy in a dynamic context raises additional interesting issues such as changes in the policymaker's reputation over time.

[27] A recent theoretical discussion of implicit bands appears in Koren (2000).

7 Appendix

Proof of Lemma 1: **(i)** We analyze the behavior of the policymaker and the speculators after the exchange rate reaches the upper bound of the band. We show that as $\varepsilon \longrightarrow 0$, there exists a unique perfect Bayesian equilibrium in which each speculator i attacks the band if and only if θ_i is above a unique threshold $\overline{\theta}^*$. The proof for the case where the exchange rate reaches the lower bound of the band is analogous.

We start with some notation. First, suppose that $x \geq \overline{\pi}$. Recalling that the policymaker defends the band if and only if $C(x, \alpha) \leq \delta$, and using equation (2.1), reveals that the policymaker will defend the band if $\overline{\pi} \leq x \leq \overline{\pi} - \alpha + \delta$, and will exit the band if $x > \overline{\pi} - \alpha + \delta$. Using these inequalities, let

$$\alpha^*(x) = \begin{cases} 0, & if \quad \overline{\pi} - x + \delta < 0, \\ \overline{\pi} - x + \delta, & if \quad 0 \leq \overline{\pi} - x + \delta \leq 1, \\ 1 & if \quad \overline{\pi} - x + \delta > 1, \end{cases} \tag{A-1}$$

be the critical measure of speculators below which the policymaker defends the upper bound of the band when the laissez faire rate of change in the exchange rate is x.

Recalling that $x \equiv \frac{e - e_{-1}}{e_{-1}}$ and $\overline{e} = (1 + \overline{\pi})e_{-1}$, the net payoffs from attacking $\overline{e}$ can be written as $(1 + x)e_{-1} - (1 + \overline{\pi})e_{-1} - t = (x - \overline{\pi})e_{-1} - t$. Since $x = \overline{\pi}$ (there is no realignment) if $\alpha < \alpha^*(x)$, and $x > \overline{\pi}$ (there is a realignment) if $\alpha \geq \alpha^*(x)$, the net payoff from attacking the upper bound of the band becomes:

$$v(x, \alpha) = \begin{cases} (x - \overline{\pi})\, e_{-1} - t, & if \quad \alpha \geq \alpha^*(x), \\ -t, & if \quad \alpha < \alpha^*(x). \end{cases} \tag{A-2}$$

Note that $\frac{\partial v(x,\alpha)}{\partial \alpha} \geq 0$ because the assumption that $x \geq \overline{\pi}$ implies that the top line in (A-2) exceeds the bottom line. Moreover, noting from (A-1) that $\frac{d\alpha^*(x)}{dx} \leq 0$, it follows that $\frac{\partial v(x,\alpha)}{\partial x} \geq 0$ with a strict inequality whenever $v(x, \alpha) \geq 0$.

Let $\alpha_i(x)$ be speculator i's belief about the measure of speculators who will attack the band for each level of x. We will say that the belief $\widehat{\alpha}_i(\cdot)$ is higher than $\alpha_i(\cdot)$ if $\widehat{\alpha}_i(\cdot) \geq \alpha_i(\cdot)$ for all x with strict inequality for at least one x.

The decision of speculator i whether or not to attack $\overline{e}$ depends on the signal θ_i that speculator i observes and his belief, $\alpha_i(\cdot)$. Using (2.4), the net expected payoffs of speculator i from attacking $\overline{e}$ is:

$$h(\theta_i, \alpha_i(\cdot)) = \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x \mid \theta_i) dx = \frac{\int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x) dx}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \tag{A-3}$$

We now establish three properties of $h(\theta_i, \alpha_i(\cdot))$:

Property 1: $h(\theta_i, \alpha_i(\cdot))$ is continuous in θ_i.

Property 2: $\widehat{\alpha}_i(\cdot) \geq \alpha_i(\cdot)$ implies that $h(\theta_i, \widehat{\alpha}_i(\cdot)) \geq h(\theta_i, \alpha_i(\cdot))$ for all θ_i.

Property 3: $\frac{\partial h(\theta_i, \alpha_i(\cdot))}{\partial \theta_i} \geq 0$ if $\alpha_i(\cdot)$ is non-decreasing in x with strict inequality whenever $h(\theta_i, \alpha_i(\cdot)) \geq 0$.

Property 1 follows because $F(\cdot)$ is a continuous function. Property 2 follows because $\frac{\partial v(x,\alpha)}{\partial \alpha} \geq 0$. To establish Property 3, note that

$$\begin{aligned}
\frac{\partial h(\theta_i, \alpha_i(\cdot))}{\partial \theta_i} &= \frac{\left[v\left(\theta_i+\varepsilon, \alpha_i(\theta_i+\varepsilon)\right) f(\theta_i+\varepsilon) - v\left(\theta_i-\varepsilon, \alpha_i(\theta_i-\varepsilon)\right) f(\theta_i-\varepsilon)\right] \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\
&\quad - \frac{\left[f(\theta_i+\varepsilon) - f(\theta_i-\varepsilon)\right] \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} v(x, \alpha_i(x)) f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\
&= \frac{f(\theta_i+\varepsilon) \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \left[v\left(\theta_i+\varepsilon, \alpha_i(\theta_i+\varepsilon)\right) - v(x, \alpha_i(x))\right] f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\
&\quad + \frac{f(\theta_i-\varepsilon) \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \left[v(x, \alpha_i(x)) - v\left(\theta_i-\varepsilon, \alpha_i(\theta_i-\varepsilon)\right)\right] f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2}.
\end{aligned} \tag{A-4}$$

Recalling that $\frac{\partial v(x,\alpha)}{\partial x} \geq 0$ and $\frac{\partial v(x,\alpha)}{\partial \alpha} \geq 0$, it follows that $\frac{\partial h(\theta_i, \alpha_i(\cdot))}{\partial \theta_i} \geq 0$ if $\alpha_i(\cdot)$ is non-decreasing in x. Moreover, (A-3) implies that if $h(\theta_i, \alpha_i(\cdot)) \geq 0$, then there exists at least one $x \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$ for which $v(x, \alpha_i(\cdot)) > 0$ (otherwise $h(\theta_i, \alpha_i(\cdot)) \leq 0$). Since $\frac{\partial v(x,\alpha)}{\partial x} > 0$ if $v(x, \alpha) \geq 0$, it follows in turn that $\frac{\partial v(x,\alpha)}{\partial x} > 0$ for at least one $x \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$. But since $\frac{\partial v(x,\alpha)}{\partial x} \geq 0$ with strict inequality for at least one x, (A-4) implies that $\frac{\partial h(\theta_i, \alpha_i(\cdot))}{\partial \theta_i} > 0$ whenever $h(\theta_i, \alpha_i(\cdot)) \geq 0$.

In equilibrium, the strategy of speculator i is to attack $\overline{e}$ if $h(\theta_i, \alpha_i(\cdot)) > 0$ and not attack it if $h(\theta_i, \alpha_i(\cdot)) < 0$. Moreover, the equilibrium belief of speculator i, $\alpha_i(\cdot)$, must be consistent with the equilibrium strategies of all other speculators (for short we will simply say that in equilibrium, "the belief of speculator i is consistent"). To characterize the equilibrium strategies of speculators, we first show that there exists a range of sufficiently large signals for which speculators have a dominant strategy to attack $\overline{e}$ and likewise, there exists a range of sufficiently small signals for which speculators have a dominant strategy not to attack $\overline{e}$. Then, we use an iterative process of elimination of dominated strategies to establish the existence of a unique signal, $\overline{\theta}^*$, such that speculator i attacks $\overline{e}$ if and only if $\theta_i > \overline{\theta}^*$.

Suppose that speculator i observes a signal $\theta_i > \overline{\theta} \equiv \overline{\pi} + \delta + \varepsilon$. Then speculator i realizes that $x > \overline{\pi} + \delta$. Using (A-1), this means that $\alpha^*(x) = 0$ so the policymaker is surely going to exit the band. By (A-2), the net payoff from attacking $\bar{e}$ is therefore $v(x,\alpha) = (x-\overline{\pi})e_{-1}-t$, for all α. But since $x > \overline{\pi}+\delta$, it follows that $v(x,\alpha) > \delta e_{-1}-t$ for all α, which is strictly positive by Assumption 2. Hence, by (A-3), $h(\theta_i, \alpha_i(x)) > 0$ for all $\theta_i > \overline{\theta}$ and all $\alpha_i(x)$, implying that it is a dominant strategy for speculator i with $\theta_i > \overline{\theta}$ to attack $\bar{e}$. Similarly, if $\theta_i < \underline{\theta} \equiv \overline{\pi}+\frac{t}{e_{-1}}-\varepsilon$ (since we focus on the case where $\varepsilon \to 0$ and since $t > 0$, such signals are observed with a positive probability whenever $x > \overline{\pi}$), speculator i realizes that $x < \overline{\pi}+\frac{t}{e_{-1}}$. Consequently, even if the policymaker surely exits the band, the payoff from attacking it is negative as $v(x,\alpha) = (x-\overline{\pi})e_{-1} - t < \left((\overline{\pi}+\frac{t}{e_{-1}}) - \overline{\pi}\right) e_{-1} - t = 0$. This implies in turn that $h(\theta_i, \alpha_i(x)) < 0$ for all $\theta_i < \underline{\theta}$ and all $\alpha_i(x)$, so it is a dominant strategy for speculator i with $\theta_i < \underline{\theta}$ not to attack.

Now, we start an iterative process of elimination of dominated strategies from $\overline{\theta}$, in order to expand the range of signals for which speculators will surely attack $\bar{e}$. To this end, let $\alpha(x,\theta)$ represent a speculator's belief regarding the measure of speculators who will attack $\bar{e}$ for each level of x, when the speculator believes that all speculators will attack $\bar{e}$ if and only if their signals are above some threshold θ. Since $\varepsilon_i \sim U[-\varepsilon,\varepsilon]$,

$$\alpha(x,\theta) = \begin{cases} 0, & if \quad x < \theta-\varepsilon, \\ \frac{x-(\theta-\varepsilon)}{2\varepsilon}, & if \quad \theta-\varepsilon \leq x \leq \theta+\varepsilon, \\ 1, & if \quad x > \theta+\varepsilon. \end{cases} \tag{A-5}$$

The iterative process of elimination of dominated strategies works as follows. Above, we already established that $h(\theta_i, \alpha_i(x)) > 0$ for all $\theta_i > \overline{\theta}$ and all $\alpha_i(x)$. But since $h(\theta_i, \alpha_i(x))$ is continuous in θ_i, it follows that $h(\overline{\theta}, \alpha_i(x)) \geq 0$ for all $\alpha_i(x)$, and in particular for $\alpha_i(x) = \alpha(x,\overline{\theta})$. Thus, $h(\overline{\theta}, \alpha(x,\overline{\theta})) \geq 0$. Note that since in equilibrium, the beliefs of speculators are consistent, only beliefs that are higher than or equal to $\alpha(x,\overline{\theta})$ can hold in equilibrium (because all speculators attack $\bar{e}$ when they observe signals above $\overline{\theta}$). Thus, we say that $\alpha(x,\overline{\theta})$ is the "lowest" consistent belief on α.

Let $\overline{\theta}^1$ be the value of θ_i for which $h(\theta_i, \alpha(x,\overline{\theta})) = 0$. That is, $h(\overline{\theta}^1, \alpha(x,\overline{\theta})) \equiv 0$. Note that $\overline{\theta}^1 \leq \overline{\theta}$, and that $\overline{\theta}^1$ is defined uniquely because we showed above that $h(\theta_i, \alpha_i(x))$ is strictly increasing in θ_i whenever $h(\theta_i, \alpha_i(x)) \geq 0$. Using Properties 2 and 3 and recalling that $\alpha(x,\overline{\theta})$ is the lowest consistent belief on α, it follows that $h(\theta_i, \alpha_i(x)) > 0$ for any $\theta_i > \overline{\theta}^1$ and any consistent belief $\alpha_i(x)$. Thus, in equilibrium, speculators must attack $\bar{e}$ if they observe signals above $\overline{\theta}^1$. As a result, $\alpha(x,\overline{\theta}^1)$ becomes the lowest consistent belief on $\alpha_i(x)$.

Starting from $\overline{\theta}^1$, we can now repeat the process along the following steps (these steps are similar to the ones that were used in order to establish $\overline{\theta}^1$). First, note that since $h(\overline{\theta}^1, \alpha(x,\overline{\theta})) \equiv 0$

and since $\alpha(x,\theta)$ is weakly decreasing with θ and $h(\theta_i, \alpha_i(x))$ is weakly increasing with $\alpha_i(x)$, it follows that $h(\overline{\theta}^1, \alpha(x, \overline{\theta}^1)) \geq 0$. Second, find a $\theta_i \leq \overline{\theta}^1$ for which $h(\theta_i, \alpha(x, \overline{\theta}^1)) = 0$, and denote it by $\overline{\theta}^2$. Using the same arguments as above, $\overline{\theta}^2$ is defined uniquely. Third, since $\alpha(x, \overline{\theta}^1)$ is the lowest consistent belief on $\alpha_i(x)$ and using the second and third properties of $h(\theta_i, \alpha_i(x)))$, it follows that speculators must attack $\overline{e}$ if they observe signals above $\overline{\theta}^2$. The lowest possible belief on $\alpha_i(x)$ becomes $\alpha(x, \overline{\theta}^2)$.

We repeat this process over and over again (each time lowering the value of θ above which speculators will attack $\overline{e}$), until we reach a step n such that $\overline{\theta}^{n+1} = \overline{\theta}^n$, implying that the process cannot continue further. Let $\overline{\theta}^\infty$ denote the value of θ at which the process stops. (Clearly, $\overline{\theta}^\infty \leq \overline{\theta}$.) By definition, speculators will attack $\overline{e}$ if they observe signals above $\overline{\theta}^\infty$. Since $\overline{\theta}^\infty$ is the point where the process stops, it must be the case that $h(\overline{\theta}^\infty, \alpha(x, \overline{\theta}^\infty)) = 0$ (otherwise, we can find some $\theta_i < \overline{\theta}^\infty$ for which $h(\theta_i, \alpha(x, \overline{\theta}^\infty)) = 0$, and the iterative process could have been continued further).

Starting a similar iterative process from $\underline{\theta}$ and following the exact same steps, we also obtain a signal $\underline{\theta}^\infty (\geq \underline{\theta})$ such that speculators will never attack $\overline{e}$ if they observe signals below $\underline{\theta}^\infty$. At this signal, it must be the case that $h(\underline{\theta}^\infty, \alpha(x, \underline{\theta}^\infty)) = 0$. Since we proved that in equilibrium speculators attack $\overline{e}$ if they observe signals above $\overline{\theta}^\infty$ and do not attack it if they observe signals below $\underline{\theta}^\infty$, it must be the case that $\overline{\theta}^\infty \geq \underline{\theta}^\infty$.

The last step of the proof involves showing that $\overline{\theta}^\infty = \underline{\theta}^\infty$ as $\varepsilon \to 0$. First, recall that $\overline{\theta}^\infty$ is defined implicitly by $h(\overline{\theta}^\infty, \alpha(x, \overline{\theta}^\infty)) = 0$. Using (A-3) and (A-5), this equality can be written as

$$\frac{\int_{\overline{\theta}^\infty - \varepsilon}^{\overline{\theta}^\infty + \varepsilon} v\left(x, \frac{x-(\overline{\theta}^\infty - \varepsilon)}{2\varepsilon}\right) f(x)dx}{F\left(\overline{\theta}^\infty + \varepsilon\right) - F\left(\overline{\theta}^\infty - \varepsilon\right)} = 0. \tag{A-6}$$

Using the equality $\alpha = \frac{x-(\overline{\theta}^\infty - \varepsilon)}{2\varepsilon}$ to change variables in the integration, (A-6) can be written as:

$$\begin{aligned} & \frac{2\varepsilon \int_0^1 v(\overline{\theta}^\infty + 2\varepsilon\alpha - \varepsilon, \alpha) f(\overline{\theta}^\infty + 2\varepsilon\alpha - \varepsilon) d\alpha}{F\left(\overline{\theta}^\infty + \varepsilon\right) - F\left(\overline{\theta}^\infty - \varepsilon\right)} \\ = & \frac{\int_0^1 v(\overline{\theta}^\infty + 2\varepsilon\alpha - \varepsilon, \alpha) f(\overline{\theta}^\infty + 2\varepsilon\alpha - \varepsilon) d\alpha}{\frac{F(\overline{\theta}^\infty + \varepsilon) - F(\overline{\theta}^\infty - \varepsilon)}{2\varepsilon}} = 0. \end{aligned} \tag{A-7}$$

As $\varepsilon \longrightarrow 0$, this equation becomes $\int_0^1 v(\overline{\theta}^\infty, \alpha)d\alpha = 0$ (by L'Hôpital's rule, the denominator approaches $f\left(\overline{\theta}^\infty\right)$ as $\varepsilon \longrightarrow 0$). Likewise, as $\varepsilon \longrightarrow 0$, $\underline{\theta}^\infty$ is defined implicitly by $\int_0^1 v(\underline{\theta}^\infty, \alpha)d\alpha = 0$. Now, assume by way of negation that $\overline{\theta}^\infty > \underline{\theta}^\infty$. Since $\frac{\partial v(x,\alpha)}{\partial x} \geq 0$ with a strict inequality when $v(x,\alpha) \geq 0$, it follows that $\int_0^1 v(\overline{\theta}^\infty, \alpha)d\alpha > \int_0^1 v(\underline{\theta}^\infty, \alpha)d\alpha$ (the strict inequality follows because

$\int_0^1 v(\overline{\theta}^{\infty}, \alpha)d\alpha = 0$ implies that $v(\overline{\theta}^{\infty}, \alpha) > 0$ for at least some values of α). This inequality contradicts the fact that $\int_0^1 v(\overline{\theta}^{\infty}, \alpha)d\alpha = 0$ and $\int_0^1 v(\underline{\theta}^{\infty}, \alpha)d\alpha = 0$.

Using the notation $\overline{\theta}^* \equiv \overline{\theta}^{\infty} = \underline{\theta}^{\infty}$, we proved that as $\varepsilon \longrightarrow 0$, there exists a unique threshold signal, $\overline{\theta}^*$, such that all speculators will attack $\overline{e}$ if and only if they observe signals above $\overline{\theta}^*$.

(ii) We now characterize $\overline{\theta}^*$. The characterization of $\underline{\theta}^*$ is then completely analogous. Suppose that $e = \overline{e}$ and suppose that absent intervention, the rate of change in the exchange rate is $x < \overline{\theta}^* - \varepsilon$. Recalling that the signals that speculators observe are drawn from the interval $[x - \varepsilon, x + \varepsilon]$, it is clear that the highest signal that a speculator can observe in this case is less than $\overline{\theta}^*$. Hence, no speculator will attack the band so $\alpha = 0$. On the other hand, if $x > \overline{\theta}^* + \varepsilon$, then the lowest signal that a speculator can observe is above $\overline{\theta}^*$. Hence, all speculators will attack $\overline{e}$ and $\alpha = 1$. In intermediate cases where $\overline{\theta}^* - \varepsilon \leq x \leq \overline{\theta}^* + \varepsilon$, some speculators will observe signals above $\overline{\theta}^*$ and will attack $\overline{e}$ while others will observe signals below $\overline{\theta}^*$ and will not attack $\overline{e}$. Given that $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$, the density of speculators who observe signals above $\overline{\theta}^*$ and attack $\overline{e}$ is $\frac{x-(\overline{\theta}^*-\varepsilon)}{2\varepsilon}$. In sum, given x and given $\overline{\theta}^*$, the measure of speculators who choose to attack $\overline{e}$ is:

$$\alpha(x, \overline{\theta}^*) = \begin{cases} 0, & x < \overline{\theta}^* - \varepsilon, \\ \frac{x-(\overline{\theta}^*-\varepsilon)}{2\varepsilon}, & \overline{\theta}^* - \varepsilon \leq x \leq \overline{\theta}^* + \varepsilon, \\ 1, & x > \overline{\theta}^* + \varepsilon. \end{cases} \tag{A-8}$$

Now, recall from above that the policymaker exits the band if $x > \overline{\pi} - \alpha + \delta$. Given $\alpha(x, \overline{\theta}^*)$, a realignment takes place if and only if $x > \overline{\pi} - \alpha(x, \overline{\theta}^*) + \delta$. Since $\alpha(x, \overline{\theta}^*)$ is weakly increasing in x, a realignment takes place if and only if x is above some threshold $\overline{x}(\overline{\theta}^*)$. Note that in a perfect Bayesian equilibrium, we cannot have $\overline{x}(\overline{\theta}^*) < \overline{\theta}^* - \varepsilon$, because then, the measure of speculators who attack the band at $x = \overline{x}(\overline{\theta}^*)$ is zero, and the policymaker should strictly prefer not to exit the band (unless $\overline{x}(\overline{\theta}^*) \geq \overline{\pi} + \delta$, but this means that $\overline{\theta}^*$ is above $\overline{\pi} + \delta + \varepsilon$, which contradicts the fact that speculators have a dominant strategy to attack the band when they observe signals above $\overline{\pi} + \delta + \varepsilon$). Similarly, one can show that we cannot have $\overline{x}(\overline{\theta}^*) > \overline{\theta}^* + \varepsilon$. Hence, in equilibrium we must have $\overline{\theta}^* - \varepsilon \leq \overline{x}(\overline{\theta}^*) \leq \overline{\theta}^* + \varepsilon$, so (A-8) implies that $\alpha(x, \overline{\theta}^*) = \frac{x-(\overline{\theta}^*-\varepsilon)}{2\varepsilon}$. Substituting into the inequality, $x > \overline{\pi} - \alpha(x, \overline{\theta}^*) + \delta$, it follows that a realignment will take place if and only if

$$x > \overline{x}(\overline{\theta}^*) \equiv \frac{\varepsilon(2\overline{\pi} + 2\delta - 1) + \overline{\theta}^*}{2\varepsilon + 1}. \tag{A-9}$$

Next, consider the decision problem that speculator i faces after observing the signal θ_i. Given that θ_i is drawn from the interval $[x - \varepsilon, x + \varepsilon]$, speculator i realizes that x is distributed on the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$, and its conditional density is given by (2.4). But, since speculator

i anticipates that the policymaker will defend $\overline{e}$ whenever $x < \overline{x}(\overline{\theta}^*)$, he expects a net payoff of $(x-\overline{\pi})e_{-1} - t$ if $x > \overline{x}(\overline{\theta}^*)$ and $-t$ if $x < \overline{x}(\overline{\theta}^*)$. Part (i) of the lemma implies that, in equilibrium, speculators attack $\overline{e}$ if and only if they observe signals above $\overline{\theta}^*$. A speculator that observes exactly $\overline{\theta}^*$ is indifferent between attacking and not attacking. Using this indifference condition, we get,

$$\int_{\overline{x}(\overline{\theta}^*)}^{\overline{\theta}^*+\varepsilon} (x-\overline{\pi})e_{-1} f(x \mid \overline{\theta}^*)dx - t = 0. \tag{A-10}$$

Substituting for $f(x \mid \cdot)$ from (2.4) into (A-10) and letting $\varepsilon \to 0$, yields:[28]

$$\lim_{\varepsilon\to 0} \frac{\int_{\overline{x}(\overline{\theta}^*)}^{\overline{\theta}^*+\varepsilon} (x-\overline{\pi})e_{-1} f(x)dx}{F\left(\overline{\theta}^*+\varepsilon\right) - F\left(\overline{\theta}^*-\varepsilon\right)} = t. \tag{A-11}$$

Substituting for $\overline{x}(\overline{\theta}^*)$ from (A-9), using L'Hôpital's rule, and recalling from (A-9) that $\overline{x}(\overline{\theta}^*) \to \overline{\theta}^*$ as $\varepsilon \to 0$, we obtain:

$$(\overline{\theta}^* - \overline{\pi})\left[1-\delta+(\overline{\theta}^*-\overline{\pi})\right] = \frac{t}{e_{-1}}. \tag{A-12}$$

Solving this equation for $\overline{\theta}^*$ yields the expression in the statement of the proposition.

(iii) Suppose that $\varepsilon \to 0$. Then, (A-9) shows that $\overline{x}(\overline{\theta}^*) \to \overline{\theta}^*$and (A-5) shows that $\alpha = 0$ if $x \le \overline{\theta}^*$ and $\alpha = 1$ if $x > \overline{\theta}^*$. Applying the same logic to the lower bound of the band, it follows that $\underline{x}(\overline{\theta}^*) \to \underline{\theta}^*$ and $\alpha = 0$ if $x \ge \underline{\theta}^*$ and $\alpha = 1$ if $x < \underline{\theta}^*$. **Q.E.D**.

Proof of Lemma 2: First, note that

$$E\pi = \int_{-\infty}^{\underline{\pi}-r} xdF(x) + \int_{\underline{\pi}-r}^{\underline{\pi}} \underline{\pi}dF(x) + \int_{\underline{\pi}}^{\overline{\pi}} xdF(x) + \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi}dF(x) + \int_{\overline{\pi}+r}^{\infty} xdF(x). \tag{A-13}$$

Moreover, note that by Assumption 1, $\frac{\partial E\pi}{\partial\overline{\pi}} = \int_{\overline{\pi}}^{\overline{\pi}+r} \left(f(x) - f(\overline{\pi}+r)\right)dx > 0$.

Next, note that sufficient conditions for a free float to be optimal is that $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial\overline{\pi}} > 0$ for all $\overline{\pi} > 0$ and $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial\underline{\pi}} < 0$ for all $\underline{\pi} < 0$. Now, suppose by way of negation that a free float is not optimal and yet $E\pi > \overline{\pi} + r$. Then, the expected payoff of the policymaker, given $\underline{\pi}$ and $\overline{\pi}$, is

$$\begin{aligned} V(\underline{\pi},\overline{\pi}) \;=\; & -A\Bigg[-\int_{-\infty}^{\underline{\pi}-r}(x-E\pi)\,dF(x) - \int_{\underline{\pi}-r}^{\underline{\pi}}(\underline{\pi}-E\pi)\,dF(x) - \int_{\underline{\pi}}^{\overline{\pi}}(x-E\pi)\,dF(x) \\ & - \int_{\overline{\pi}}^{\overline{\pi}+r}(\overline{\pi}-E\pi)\,dF(x) - \int_{\overline{\pi}+r}^{E\pi}(x-E\pi)\,dF(x) + \int_{E\pi}^{\infty}(x-E\pi)\,dF(x)\Bigg] \\ & - \int_{-\infty}^{\underline{\pi}-r}\delta dF(x) - \int_{\underline{\pi}-r}^{\underline{\pi}}(\underline{\pi}-x)dF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r}(x-\overline{\pi})dF(x) - \int_{\overline{\pi}+r}^{\infty}\delta dF(x). \end{aligned} \tag{A-14}$$

[28] Note that when ε tends to zero both numerator and denominator in equation (3.5) go to zero.

Differentiating with respect to $\overline{\pi}$ yields:

$$\begin{aligned}\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}} &= -A\left[(2F(E\pi)-1)\frac{\partial E\pi}{\partial \overline{\pi}} - \int_{\overline{\pi}}^{\overline{\pi}+r}(f(x)-f(\overline{\pi}+r))\,dx\right] \\ &\quad + \int_{\overline{\pi}}^{\overline{\pi}+r}(f(x)-f(\overline{\pi}+r))\,dx + \delta f(\overline{\pi}+r) \qquad \text{(A-15)} \\ &= (2A(1-F(E\pi))+1)\frac{\partial E\pi}{\partial \overline{\pi}} + \delta f(\overline{\pi}+r) > 0,\end{aligned}$$

where we used the fact that, due to Assumption 1, $\frac{\partial E\pi}{\partial \overline{\pi}} > 0$. Hence $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}} > 0$ for all $\overline{\pi} > 0$, thus contradicting the assumption that a free float is not optimal.

Next, suppose by way of negation that a free float is not optimal and yet $E\pi \in [\overline{\pi}, \overline{\pi}+r]$. Then, the expected payoff of the policymaker, given $\underline{\pi}$ and $\overline{\pi}$, is

$$\begin{aligned}V(\underline{\pi},\overline{\pi}) &= -A\left[-\int_{-\infty}^{\underline{\pi}-r}(x-E\pi)\,dF(x) - \int_{\underline{\pi}-r}^{\underline{\pi}}(\underline{\pi}-E\pi)\,dF(x) - \int_{\underline{\pi}}^{\overline{\pi}}(x-E\pi)\,dF(x)\right. \\ &\quad \left. - \int_{\overline{\pi}}^{\overline{\pi}+r}(\overline{\pi}-E\pi)\,dF(x) + \int_{\overline{\pi}+r}^{\infty}(x-E\pi)\,dF(x)\right] \qquad \text{(A-16)} \\ &\quad - \int_{-\infty}^{\underline{\pi}-r}\delta dF(x) - \int_{\underline{\pi}-r}^{\underline{\pi}}(\underline{\pi}-x)dF(x) - \int_{\overline{\pi}}^{\overline{\pi}+r}(x-\overline{\pi})dF(x) - \int_{\overline{\pi}+r}^{\infty}\delta dF(x).\end{aligned}$$

Using again the fact that $\frac{\partial E\pi}{\partial \overline{\pi}} > 0$, we get:

$$\begin{aligned}\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}} &= -A\left[(2F(\overline{\pi}+r)-1)\frac{\partial E\pi}{\partial \overline{\pi}} + 2(E\pi-\overline{\pi})f(\overline{\pi}+r) - rf(\overline{\pi}+r) - \int_{\overline{\pi}}^{\overline{\pi}+r}dF(x)\right] \\ &\quad + \int_{\overline{\pi}}^{\overline{\pi}+r}(f(x)-f(\overline{\pi}+r))\,dx + \delta f(\overline{\pi}+r) \\ &\geq -A\left[\frac{\partial E\pi}{\partial \overline{\pi}} + rf(\overline{\pi}+r) - \int_{\overline{\pi}}^{\overline{\pi}+r}f(x)dx\right] + \frac{\partial E\pi}{\partial \overline{\pi}} + \delta f(\overline{\pi}+r) \qquad \text{(A-17)} \\ &= -A\left[\frac{\partial E\pi}{\partial \overline{\pi}} - \int_{\overline{\pi}}^{\overline{\pi}+r}(f(x)-f(\overline{\pi}+r))\,dx\right] + \frac{\partial E\pi}{\partial \overline{\pi}} + \delta f(\overline{\pi}+r) \\ &= \frac{\partial E\pi}{\partial \overline{\pi}} + \delta f(\overline{\pi}+r) > 0,\end{aligned}$$

where the inequality follows because (i) $F(\overline{\pi}+r) < 1$ and (ii) $E\pi \in [\overline{\pi}, \overline{\pi}+r]$ implies that $E\pi - \overline{\pi} < r$. Hence once again $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}} > 0$ for all $\overline{\pi} > 0$, thus contradicting the assumption that a free float is not optimal.

The proofs that whenever $E\pi < \underline{\pi} - r$ or $E\pi \in [\underline{\pi} - r, \underline{\pi}]$, a free float must be optimal are analogous. **Q.E.D.**

Lemma A1: *Jointly sufficient conditions for $V(\underline{\pi},\overline{\pi})$ to be globally concave are*

$$A > Max\left\{\frac{1}{2(1-F(E\overline{\pi}))}, \frac{1}{2F(E\overline{\pi})}\right\}$$

and $f"(\cdot) \leq 0$ *for all* x.

Proof of Lemma A1: Using (3.4),

$$\frac{\partial^2 V(\underline{\pi}, \overline{\pi})}{\partial \overline{\pi}^2} = -2Af(E\pi)\left(\frac{\partial E\pi}{\partial \overline{\pi}}\right)^2 + (1 - 2AF(E\pi))\frac{\partial^2 E\pi}{\partial \overline{\pi}^2} + \delta f'(\overline{\pi} + r), \qquad \text{(A-18)}$$

where given that $f"(\cdot) \leq 0$, it follows from (A-13) that

$$\frac{\partial^2 E\pi}{\partial \overline{\pi}^2} = f(\overline{\pi} + r) - f(\overline{\pi}) - rf'(\overline{\pi} + r) \leq 0. \qquad \text{(A-19)}$$

Together with $A > \frac{1}{2F(E\pi)}$, (A-19) implies that the second term in (A-18) is nonpositive. By Assumption 1, $f'(\overline{\pi} + r) < 0$. Hence, $\frac{\partial^2 V(\underline{\pi}, \overline{\pi})}{\partial \overline{\pi}^2} < 0$. The proof that $\frac{\partial^2 V(\underline{\pi}, \overline{\pi})}{\partial \underline{\pi}^2} < 0$ is analogous.

Using (3.4), the cross partial derivative of $V(\underline{\pi}, \overline{\pi})$ is

$$\begin{aligned}
\frac{\partial^2 V(\underline{\pi}, \overline{\pi})}{\partial \underline{\pi} \partial \overline{\pi}} &= -2A\left[f(E\pi)\frac{\partial E\pi}{\partial \overline{\pi}}\frac{\partial E\pi}{\partial \underline{\pi}} + F(E\pi)\frac{\partial^2 E\pi}{\partial \underline{\pi} \partial \overline{\pi}}\right] + \frac{\partial^2 E\pi}{\partial \underline{\pi} \partial \overline{\pi}} \\
&= -2Af(E\pi)\frac{\partial E\pi}{\partial \overline{\pi}}\frac{\partial E\pi}{\partial \underline{\pi}} < 0, \qquad \text{(A-20)}
\end{aligned}$$

where the second equality follows because $\frac{\partial^2 E\pi}{\partial \underline{\pi} \partial \overline{\pi}} = 0$. The determinant of the Hessian matrix is given by

$$\begin{aligned}
H(\cdot) &\equiv \frac{\partial^2 V(\underline{\pi}, \overline{\pi})}{\partial \underline{\pi}^2}\frac{\partial^2 V(\underline{\pi}, \overline{\pi})}{\partial \overline{\pi}^2} - \left(\frac{\partial^2 V(\underline{\pi}, \overline{\pi})}{\partial \underline{\pi} \partial \overline{\pi}}\right)^2 \\
&= -\frac{\partial^2 E\pi}{\partial \overline{\pi}^2}(1 - 2AF(E\pi))\left[2Af(E\pi)\left(\frac{\partial E\pi}{\partial \underline{\pi}}\right)^2 + \delta f'(\overline{\pi} - r)\right] \\
&\quad + \frac{\partial^2 E\pi}{\partial \underline{\pi}^2}(1 - 2A(1 - F(E\pi)))\left[2Af(E\pi)\left(\frac{\partial E\pi}{\partial \overline{\pi}}\right)^2 - \delta f'(\overline{\pi} + r)\right] \qquad \text{(A-21)} \\
&\quad - \frac{\partial^2 E\pi}{\partial \overline{\pi}^2}\frac{\partial^2 E\pi}{\partial \underline{\pi}^2}((1 - 2AF(E\pi)))(1 - 2A(1 - F(E\pi))) - \delta^2 f'(\overline{\pi} + r)f'(\overline{\pi} - r) \\
&\quad + 2A\delta f(E\pi)\left[\left(\frac{\partial E\pi}{\partial \overline{\pi}}\right)^2 f'(\overline{\pi} - r) - \left(\frac{\partial E\pi}{\partial \underline{\pi}}\right)^2 f'(\overline{\pi} + r)\right].
\end{aligned}$$

By Assumption 1, $f'(\overline{\pi} - r) > 0 > f'(\overline{\pi} + r)$. The condition $A > Max\left\{\frac{1}{2(1-F(E\pi))}, \frac{1}{2F(E\pi)}\right\}$ implies that $1 - 2AF(E\pi) < 0$ and $1 - 2A(1 - F(E\pi) < 0$, and $f"(\cdot) \leq 0$ for all x implies that $\frac{\partial^2 E\pi}{\partial \overline{\pi}^2} \geq 0$ and $\frac{\partial^2 E\pi}{\partial \underline{\pi}^2} \leq 0$. Hence, under these conditions, $H(\cdot) > 0$, implying that $V(\underline{\pi}, \overline{\pi})$ is globally concave. **Q.E.D.**

Proof of Lemma 3: We begin by showing that if the policymaker's problem has internal solutions, then at least one of them must be symmetric. Specifically, we will show that if $(\underline{\pi}^*, -\underline{\pi}^*)$ satisfies

$\frac{\partial V(\underline{\pi}^*,-\underline{\pi}^*)}{\partial \underline{\pi}} = 0$, then it also satisfies $\frac{\partial V(\underline{\pi}^*,-\underline{\pi}^*)}{\partial \overline{\pi}} = 0$. To this end, note that since $f(x)$ is symmetric around 0, then for $0 < a < b$,

$$f(x) = f(-x), \quad and \quad \int_{-b}^{-a} f(x)dx = \int_{a}^{b} f(x)dx.$$

Since $\overline{\pi}^* = -\underline{\pi}^*$ and since $f(x)$ is symmetric around 0, then $E\pi = 0$ and $F(E\pi) = 1/2$. Using these facts in equation (3.4) we obtain

$$\begin{aligned} \frac{\partial V(\underline{\pi}^*,-\underline{\pi}^*)}{\partial \overline{\pi}} &= (1-A)\int_{-\underline{\pi}^*}^{-\underline{\pi}^*+r} \left(f(x) - f(-\underline{\pi}^*+r)\right)dx + \delta f(-\underline{\pi}^*+r) \\ &= (1-A)\int_{\underline{\pi}^*-r}^{\underline{\pi}^*} \left(f(x) - f(\underline{\pi}^*-r)\right)dx + \delta f(\underline{\pi}^*-r), \end{aligned} \tag{A-22}$$

where the second equality follows from the relations immediately preceding (A-22). But the last line in equation (A-22) vanishes because the first order condition for an interior solution for $\underline{\pi}$ implies that whenever $\overline{\pi} = -\underline{\pi}^*$ (so that $E\pi = 0$ and $F(E\pi) = 1/2$),

$$\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}} = -(1-A)\int_{\underline{\pi}^*-r}^{\underline{\pi}^*} \left(f(x) - f(\underline{\pi}^*-r)\right)dx - \delta f(\underline{\pi}^*-r) = 0. \tag{A-23}$$

Hence, $\frac{\partial V(\underline{\pi}^*,-\underline{\pi}^*)}{\partial \overline{\pi}} = 0$ as required.

We now show that there are no asymmetric solutions to the policymaker's problem. To this end, pick an internal symmetric solution, $(-\overline{\pi}^*, \overline{\pi}^*)$, so that

$$\frac{\partial V(-\overline{\pi}^*,\overline{\pi}^*)}{\partial \overline{\pi}} = 0, \qquad and \qquad \frac{\partial V(-\overline{\pi}^*,\overline{\pi}^*)}{\partial \underline{\pi}} = 0. \tag{A-24}$$

Now, fix the upper bound at $\overline{\pi}^*$ and lower the lower bound below $-\overline{\pi}^*$. Due to the symmetry of $f(x)$, $E\pi = 0$ at $(-\overline{\pi}^*, \overline{\pi}^*)$ and $E\pi < 0$ at $(\underline{\pi}, \overline{\pi}^*)$. Hence, $F(E\pi) < \frac{1}{2}$ at $(\underline{\pi}, \overline{\pi}^*)$, so by (3.4),

$$\begin{aligned} \frac{\partial V(\underline{\pi},\overline{\pi}^*)}{\partial \overline{\pi}} &= (1-2AF(E\pi))\int_{\overline{\pi}^*}^{\overline{\pi}^*+r} \left(f(x) - f(\overline{\pi}^*+r)\right)dx + \delta f(\overline{\pi}^*+r) \\ &> (1-A)\int_{\overline{\pi}^*}^{\overline{\pi}^*+r} \left(f(x) - f(\overline{\pi}^*+r)\right)dx + \delta f(\overline{\pi}^*+r) \\ &= \frac{\partial V(-\overline{\pi}^*,\overline{\pi}^*)}{\partial \overline{\pi}} = 0, \end{aligned} \tag{A-25}$$

where the last two equalities follow by using the fact that $E\pi = 0$ at $(-\overline{\pi}^*, \overline{\pi}^*)$ in equation (3.4). Since $\frac{\partial V(\underline{\pi},\overline{\pi}^*)}{\partial \overline{\pi}} \neq 0$, $(\underline{\pi}, \overline{\pi}^*)$ is not an internal maximum. The proof for the case $\underline{\pi} > -\overline{\pi}^*$ is analogous.

Since both $f(x)$ and the band are symmetric, $E\pi = 0$, so that appreciations and depreciations are equally likely. **Q.E.D.**

Proof of Proposition 2: Since both the band and $f(x)$ are symmetric around 0, $E\pi = 0$. Hence, equations (3.3) and (3.4) become:

$$\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}} = -(1-A)\int_{\underline{\pi}-r}^{\underline{\pi}} (f(x) - f(\underline{\pi}-r))\,dx - \delta f(\underline{\pi}-r) = 0, \tag{3.3a}$$

and

$$\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}} = (1-A)\int_{\overline{\pi}}^{\overline{\pi}+r} (f(x) - f(\overline{\pi}+r))\,dx + \delta f(\overline{\pi}+r) = 0. \tag{3.4a}$$

Note that since $E\pi = 0$ and since $f(x)$ is symmetric, $F(E\pi) = \frac{1}{2}$, the condition $A > Max\left\{\frac{1}{2(1-F(E\pi))}, \frac{1}{2F(E\pi)}\right\}$ reduces to $A > 1$. Hence, $A > 1$, along with the assumption that $f"(x) \leq 0$ for all x and with Assumption 1, are sufficient (but not necessary) conditions for $V(\underline{\pi},\overline{\pi})$ to be globally concave, in which case (3.3a) and (3.4a) are sufficient for a unique maximum.

(i) Assumption 1 implies that $f(x) > f(\underline{\pi}-r)$ for all $x \in [\underline{\pi}-r, \underline{\pi}]$. Hence, if $A \leq 1$, then $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}} < 0$ for all $\underline{\pi} < 0$, implying that the policymaker will push $\underline{\pi}$ all the way to $-\infty$. Likewise, Assumption 1 implies that $f(x) > f(\overline{\pi}+r)$ for all $x \in [\overline{\pi}, \overline{\pi}+r]$; if $A \leq 1$, then $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}} > 0$ for all $\overline{\pi} > 0$, implying that the policymaker will push $\overline{\pi}$ all the way to ∞.

(ii) To establish that $\underline{\pi} < 0$, it is sufficient to show that evaluated at $\underline{\pi} = 0$, $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}} < 0$ (the policymaker will not push $\underline{\pi}$ all the way up to 0). Using (3.3a) we obtain that

$$\begin{aligned}\left.\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}}\right|_{\underline{\pi}=0} &= -\delta f(-r) + (A-1)\int_{-r}^{0} [f(x) - f(-r)]\,dx \\ &= -\int_{-r}^{0} [f(x) - f(-r)]\,dx \left[\frac{\delta}{\int_{-r}^{0}\left[\frac{f(x)}{f(-r)} - 1\right]dx} + 1 - A\right].\end{aligned} \tag{A-26}$$

By Assumption 1, the integral term outside the square brackets in the second line of (A-26) is negative. By the symmetry of $f(x)$, $\int_{-r}^{0}\left[\frac{f(x)}{f(-r)} - 1\right]dx = \int_{0}^{r}\left[\frac{f(x)}{f(r)} - 1\right]dx$. Hence, if $A < \overline{A}(r)$, the square bracketed term is positive, so it is optimal to set $\underline{\pi} < 0$. To show that $\overline{\pi} > 0$, it is sufficient to show that evaluated at $\overline{\pi} = 0$, $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}} > 0$ (the policymaker will increases $\overline{\pi}$ above 0). Using (3.4a) we obtain that,

$$\begin{aligned}\left.\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}}\right|_{\overline{\pi}=0} &= \delta f(r) + (1-A)\int_{0}^{r} [f(x) - f(r)]\,dx \\ &= \int_{0}^{r} [f(x) - f(r)]\,dx \left[\frac{\delta}{\int_{0}^{r}\left[\frac{f(x)}{f(r)} - 1\right]dx} + 1 - A\right].\end{aligned} \tag{A-27}$$

By Assumption 1, the integral term outside the square brackets in the second line of (A-27) is positive. Since $A < \overline{A}(r)$, the square bracketed term is positive too. Hence, it is optimal to set

$\overline{\pi} > 0$. Since $A > 1$, it follows from part (i) that the exchange rate regime is not a free float. Hence, $-\infty < \underline{\pi} < 0 < \overline{\pi} < \infty$.

(iii) Since $f''(x) \leq 0$ and $A > 1$, $V(\underline{\pi}, \overline{\pi})$ is concave in both $\underline{\pi}$ and $\overline{\pi}$. It follows that a sufficient condition for $\underline{\pi} = 0$ is that, evaluated at $\underline{\pi} = 0$, $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}} \geq 0$ (the policymaker would like to push $\underline{\pi}$ all the way up to 0). From part (ii) of the proposition it is obvious that this occurs when $A > \overline{A}(r)$. Likewise, since $V(\underline{\pi}, \overline{\pi})$ is concave in $\overline{\pi}$, then a sufficient condition for $\overline{\pi} = 0$ is that, evaluated at $\overline{\pi} = 0$, $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}} \leq 0$ (the policymaker would not like to increase $\overline{\pi}$ above 0). From part (ii) of the proposition it is obvious that this is the case when $A > \overline{A}(r)$. **Q.E.D**.

Proof of Proposition 3: (i) Proposition 1 implies that r decreases when t decreases. Straightforward differentiation of (3.3a) and (3.4a) and use of Assumption 1 show that $\underline{\pi}$ decreases and $\bar{\pi}$ increases when r decreases. Hence a decrease in t leads to a decrease in $\underline{\pi}$ and to an increase in $\bar{\pi}$.

By Lemma 1, the probability of a speculative attack is $P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r))$. Now,

$$\begin{aligned} \frac{\partial P}{\partial t} &= f(\underline{\pi} - r)\left[\frac{\partial \underline{\pi}}{\partial r}\frac{\partial r}{\partial t} - \frac{\partial r}{\partial t}\right] - f(\overline{\pi} + r)\left[\frac{\partial \overline{\pi}}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial r}{\partial t}\right] \\ &= \left[f(\underline{\pi} - r)\left[\frac{\partial \underline{\pi}}{\partial r} - 1\right] - f(\overline{\pi} + r)\left[\frac{\partial \overline{\pi}}{\partial r} + 1\right]\right]\frac{\partial r}{\partial t}. \end{aligned} \tag{A-28}$$

By Proposition 1, $\frac{\partial r}{\partial t} > 0$. Hence it is sufficient to establish that $\frac{\partial \underline{\pi}}{\partial r} > 1$ and $\frac{\partial \overline{\pi}}{\partial r} < -1$. Using (3.3a), it follows that

$$\frac{\partial \underline{\pi}}{\partial r} = \frac{\frac{\partial^2 V(\underline{\pi},\overline{\pi})}{\partial r \partial \underline{\pi}}}{\frac{\partial^2 V(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}^2}} = \frac{-\left((A-1)r + \delta\right) f'(\underline{\pi} - r)}{-\left[r(A-1) + \delta\right] f'(\underline{\pi} - r) + (A-1)\left[f(\underline{\pi}) - f(\underline{\pi} - r)\right]}, \tag{A-29}$$

which exceeds 1 because of Assumption 1 and because $A > 1$. Likewise, using (3.4a), it follows that,

$$\frac{\partial \overline{\pi}}{\partial r} = \frac{\frac{\partial^2 V}{\partial r \partial \overline{\pi}}}{\frac{\partial^2 V}{\partial \overline{\pi}^2}} = \frac{-\left((A-1)r + \delta\right) f'(\overline{\pi} + r)}{-\left((A-1)r + \delta\right) f'(\overline{\pi} + r) + (A-1)\left(f(\overline{\pi} + r) - f(\overline{\pi})\right)}, \tag{A-30}$$

which is less than -1 due to Assumption 1 and because $A > 1$.

(ii) Differentiating $\overline{A}(r)$ with respect to t yields:

$$\frac{\partial \overline{A}(r)}{\partial t} = \frac{\delta f'(r) \int_0^r \frac{f(x)}{f(r)^2} dx}{\left(\int_0^r \left[1 - \frac{f(x)}{f(r)}\right] dx\right)^2} \frac{\partial r}{\partial t} < 0,$$

where the inequality follow because $\frac{\partial r}{\partial t} > 0$ by Proposition 1, and $f'(r) < 0$ by Assumption 1. Part (iii) of Proposition 2 implies that the policymaker prefers to set a peg when $A > \overline{A}(r)$. Since $\overline{A}(r)$ falls with t, this condition is more likely to hold when t is larger.

(iii) Using (3.2) for the case $E\pi = 0$ and the envelope theorem it follows that:

$$\frac{\partial V(\underline{\pi}, \overline{\pi})}{\partial t} = (r(A-1) + \delta)\left[f(\underline{\pi} - r) + f(\overline{\pi} + r)\right] \frac{\partial r}{\partial t} > 0, \tag{A-31}$$

where the inequality follows from the restriction $A > 1$ and because by Proposition 1, $\frac{\partial r}{\partial t} > 0$. **Q.E.D**

Proof of Proposition 4: **(i)** The proof follows by straightforward differentiation of (3.3a) and (3.4a) and by using Assumption 1.

(ii) By Lemma 1, the probability of a speculative attack is $P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r))$. Straightforward differentiation of P with respect to A along with part (i) of the proposition establish the result. **Q.E.D.**

Proof of Proposition 5: Let $\underline{\pi}^f$ and $\overline{\pi}^f$ be the solutions to the policymaker's maximization problem when the density function is $f(x)$ and let $\underline{\pi}^g$ and $\overline{\pi}^g$ be the corresponding solutions when the density function is $g(x)$. $\underline{\pi}^g$ is defined by (3.3a) with $g(x)$ replacing $f(x)$. Now, let's evaluate $\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}}$ when the density is $g(x)$ at $\underline{\pi}^f$:

$$\begin{aligned}
\left.\frac{\partial V(\underline{\pi}, \overline{\pi})}{\partial \underline{\pi}}\right|_{\underline{\pi}=\underline{\pi}^f} &= -\delta g(\underline{\pi}^f - r) + (A-1)\int_{\underline{\pi}^f - r}^{\underline{\pi}^f} \Big[g(x) - g(\underline{\pi}^f - r)\Big]\, dx \\
&= -\delta f(\underline{\pi}^f - r) + (A-1)\int_{\underline{\pi}^f - r}^{\underline{\pi}^f} \Big[g(x) - f(\underline{\pi}^f - r)\Big]\, dx \\
&< -\delta f(\underline{\pi}^f - r) + (A-1)\int_{\underline{\pi}^f - r}^{\underline{\pi}^f} \Big[f(x) - f(\underline{\pi}^f - r)\Big]\, dx = 0,
\end{aligned} \tag{A-32}$$

where the first equality follows because by assumption, $g(\underline{\pi}^f - r) = f(\underline{\pi}^f - r)$. The inequality follows because $f(x)$ lies above $g(x)$ whenever $x > \underline{\pi}^f - r$, and the second equality follows from (3.3a). Since $\left.\frac{\partial V(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}}\right|_{\underline{\pi}=\underline{\pi}^f} < 0$, it follows that $\underline{\pi}^f > \underline{\pi}^g$. The proof that $\overline{\pi}^f < \overline{\pi}^g$ is analogous. Hence, $\underline{\pi}^g < \underline{\pi}^f < \overline{\pi}^f < \overline{\pi}^g$, so the band becomes wider under $g(x)$. **Q.E.D.**

Proof of Lemma 4: **(i)** Suppose the exchange rate reaches the upper bound of the band. Given x, a dependable policymaker will defend the upper bound of the band if and only if $\alpha < \alpha^*(x)$, where $\alpha^*(x)$ is given by (A-1). Hence, the net payoff from attacking the upper bound of the band is:

$$v^{\beta}(x, \alpha) = \begin{cases} (x - \overline{\pi})\, e_{-1} - t, & if \quad \alpha \geq \alpha^*(x), \\ (1-\beta)(x - \overline{\pi})\, e_{-1} - t, & if \quad \alpha < \alpha^*(x). \end{cases}$$

Since $v^{\beta}(x,\alpha)$ has the same properties as $v(x,\alpha)$ defined by (A-2), the equilibrium analysis here is exactly as in the proof of part (i) of Lemma 1. Hence, once again we have a unique equilibrium in which speculators attack the upper bound of the band if and only if they observe a signal above a unique threshold, $\overline{\theta}^*_{\beta}$.

We turn next to a characterization of the behavior of the dependable policymaker in equilibrium. Since in equilibrium, $\alpha(x)$ is increasing in x ($\alpha(x)$ is given by (A-8), where $\overline{\theta}^*_{\beta}$ replaces $\overline{\theta}^*$), and since $C(x,\alpha(x))$ is increasing in both x and $\alpha(x)$, then the dependable policymaker will exit the band if and only if x is above some threshold level, $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$.

In order to establish that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) \to \overline{\theta}^*_{\beta}$ as $\varepsilon \to 0$, we now show that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$ must be in the interval $\left[\overline{\theta}^*_{\beta}-\varepsilon, \overline{\theta}^*_{\beta}+\varepsilon\right]$. In order to see this, suppose by way of negation that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) > \overline{\theta}^*_{\beta}+\varepsilon$. Then, speculators who observe $\overline{\theta}^*_{\beta}$ know that a dependable policymaker will defend the band. Thus, the payoff they expect to get from attacking the band is lower than $(1-\beta)\left(\overline{\theta}^*_{\beta}+\varepsilon-\overline{\pi}\right)e_{-1}-t$. By equilibrium conditions and continuity, we know that speculators who observe $\overline{\theta}^*_{\beta}$ must be indifferent between attacking the band and not attacking it. This means that $(1-\beta)\left(\overline{\theta}^*_{\beta}+\varepsilon-\overline{\pi}\right)e_{-1}-t>0$. However, using Assumption 4, this condition will hold only if $\overline{\theta}^*_{\beta}+\varepsilon > \overline{\pi}+\delta$. Since by assumption, $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) > \overline{\theta}^*_{\beta}+\varepsilon$, this implies in turn that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) > \overline{\pi}+\delta$, thereby contradicting the fact that a dependable policymaker always exits the band when $x > \overline{\pi}+\delta$. Thus, $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$ cannot be above $\overline{\theta}^*_{\beta}+\varepsilon$. Next, suppose by way of negation that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) < \overline{\theta}^*_{\beta}-\varepsilon$. Then, at $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$, a dependable policymaker knows that no speculator attacks the band. By equilibrium conditions and continuity, at $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$, a dependable policymaker must be indifferent between existing the band and maintaining it, that is, $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) = \overline{\pi}+\delta$. Since by assumption, $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) < \overline{\theta}^*_{\beta}-\varepsilon$, this means that $\overline{\theta}^*_{\beta} > \overline{\pi}+\delta+\varepsilon$, which contradicts the fact that speculators have a dominant strategy to attack the band when they observe signals above $\overline{\pi}+\delta+\varepsilon$. Thus, $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$ cannot be below $\overline{\theta}^*_{\beta}-\varepsilon$

Given the fact that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$ is in the interval $\left[\overline{\theta}^*_{\beta}-\varepsilon, \overline{\theta}^*_{\beta}+\varepsilon\right]$ and using (A-8), it follows that:

$$\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) = \frac{\varepsilon(2\overline{\pi}+2\delta-1)+\overline{\theta}^*_{\beta}}{2\varepsilon+1}. \tag{A-33}$$

Since a speculator that observes $\overline{\theta}^*_{\beta}$ is indifferent between attacking the band and not attacking it, the equation that defines $\overline{\theta}^*_{\beta}$ is given by:

$$\beta\int_{\overline{x}_{\beta}(\overline{\theta}^*_{\beta})}^{\overline{\theta}^*_{\beta}+\varepsilon}(x-\overline{\pi})e_{-1}f(x\mid\overline{\theta}^*_{\beta})dx+(1-\beta)\int_{\overline{\theta}^*_{\beta}-\varepsilon}^{\overline{\theta}^*_{\beta}+\varepsilon}(x-\overline{\pi})e_{-1}f(x\mid\overline{\theta}^*_{\beta})dx=t, \tag{A-34}$$

where $f(x\mid\cdot)$ is defined by (2.4). This equation coincides with equation (A-10) if $\beta=1$. Substituting from (2.4) for $f(x \mid \cdot)$ into (A-34), and taking the limit as $\varepsilon \to 0$, yields:

$$\lim_{\varepsilon \to 0} \frac{\beta \int_{\overline{x}_\beta(\overline{\theta}^*_\beta)}^{\overline{\theta}^*_\beta+\varepsilon} (x-\overline{\pi})e_{-1} f(x)dx + (1-\beta)\int_{\overline{\theta}^*_\beta-\varepsilon}^{\overline{\theta}^*_\beta+\varepsilon} (x-\overline{\pi})e_{-1} f(x)dx}{F\left(\overline{\theta}^*_\beta+\varepsilon\right) - F(\overline{\theta}^*_\beta-\varepsilon)} = t. \tag{A-35}$$

Using L'Hôpital's rule, and the expression for $\overline{x}_\beta(\overline{\theta}^*_\beta)$, and recalling that $\overline{x}_\beta(\overline{\theta}^*_\beta) \to \overline{\theta}^*_\beta$ as $\varepsilon \to 0$, we obtain:

$$(\overline{\theta}^*_\beta - \overline{\pi})(1-\beta\delta+\beta(\overline{\theta}^*_\beta-\overline{\pi}))e_{-1} = t. \tag{A-36}$$

Solving this equation for $\overline{\theta}^*_\beta$ reveals that $\overline{\theta}^*_\beta = \overline{\pi} + r^\beta$, where r^β is defined by (4.1). Using similar arguments, it follows that $\underline{\theta}^*_\beta = \underline{\pi} - r^\beta$.

(ii) Differentiating r^β with respect to β we obtain:

$$\frac{\partial r^\beta}{\partial \beta} = \frac{1}{2\beta^2}\left[1 - \frac{\frac{t}{e_{-1}} - \frac{\delta-\frac{1}{\beta}}{2}}{\sqrt{\frac{t}{\beta e_{-1}} + \frac{(\delta-\frac{1}{\beta})^2}{4}}}\right]. \tag{A-37}$$

This derivative is positive if and only if the expression inside the brackets is positive. This is the case, in turn, if and only if

$$\frac{t}{\beta e_{-1}} + \frac{(\delta-\frac{1}{\beta})^2}{4} > \left[\frac{t}{e_{-1}} - \frac{\delta-\frac{1}{\beta}}{2}\right]^2. \tag{A-38}$$

Further rearrangement of the last inequality shows that it is equivalent to Assumption 4. Hence, r^β increases with β. **Q.E.D.**

Proof of Proposition 6: Since $f(x)$ is symmetric by Assumption 3, the first order conditions for an interior solution for the problem of a dependable policymaker are:

$$\begin{aligned}\frac{\partial V^\beta(\underline{\pi},\overline{\pi})}{\partial \underline{\pi}} &= -\left[r^\beta(A\beta-1)+\delta\right] f(\underline{\pi}-r^\beta) + (A\beta-1)\int_{\underline{\pi}-r^\beta}^{\underline{\pi}} f(x)dx \\ &= A\beta\int_{\underline{\pi}-r^\beta}^{\underline{\pi}}\left[f(x)-f(\underline{\pi}-r^\beta)\right]dx - \left[\int_{\underline{\pi}-r^\beta}^{\underline{\pi}}\left[f(x)-f(\underline{\pi}-r^\beta)\right]dx + \delta f(\underline{\pi}-r^\beta)\right] = 0,\end{aligned} \tag{A-39}$$

and,

$$\begin{aligned}\frac{\partial V^\beta(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}} &= \left[r^\beta(A\beta-1)+\delta\right] f(\overline{\pi}+r^\beta) - (A\beta-1)\int_{\overline{\pi}}^{\overline{\pi}+r^\beta} f(x)dx \\ &= -A\beta\int_{\overline{\pi}}^{\overline{\pi}+r^\beta}\left[f(x)-f(\overline{\pi}+r^\beta)\right]dx + \left[\int_{\overline{\pi}}^{\overline{\pi}+r^\beta}\left[f(x)-f(\overline{\pi}+r^\beta)\right]dx + \delta f(\overline{\pi}+r^\beta)\right] = 0.\end{aligned} \tag{A-40}$$

As in the case where $\beta < 1$, it can be shown that $f''(x) \le 0$ for all $x's$ and $A\beta > 1$, along with Assumption 1, are sufficient for $V^\beta((\underline{\pi},\overline{\pi})$ to be globally concave in $\underline{\pi}$ and $\overline{\pi}$ so (A-39) and (A-40) are sufficient for a unique maximum.

(i)-(iii) The proofs follow the corresponding proofs in Proposition 2 with r replaced by r^{β}.

(iv) Differentiating $\frac{\partial V^{\beta}(\underline{\pi},\overline{\pi})}{\partial \overline{\pi}}$ with respect to β and $\overline{\pi}$, and using the implicit function theorem,yields:

$$\frac{\partial \overline{\pi}}{\partial \beta} = -\frac{A\int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} \left[f(\overline{\pi}+r^{\beta}) - f(x)\right] dx + \left[r^{\beta}(A\beta-1)+\delta\right] f'(\overline{\pi}+r^{\beta})\frac{\partial r^{\beta}}{\partial \beta}}{-(A\beta-1)\left[f(\overline{\pi}+r^{\beta}) - f(\overline{\pi})\right] + \left[r^{\beta}(A\beta-1)+\delta\right] f'(\overline{\pi}+r^{\beta})}. \quad \text{(A-41)}$$

Assumption 1 ensures that the integral term in the numerator is negative and it also ensures that $f'(\overline{\pi}+r^{\beta}) < 0$. Since by Lemma 4, $\frac{\partial r^{\beta}}{\partial \beta} > 0$, it follows that the numerator is negative. By the second order conditions for maximization, the denominator is negative so $\overline{\pi}$ decreases towards 0. Similarly, it can be shown that as β increases, $\underline{\pi}$ increases towards 0. Hence, an increase in β leads to a tighter band.

Finally, the probability of a speculative attack is now $P^{\beta} = F(\underline{\pi} - r^{\beta}) + (1 - F(\overline{\pi}+r^{\beta}))$. Differentiating this expression with respect to β yields:

$$\frac{\partial P^{\beta}}{\partial \beta} = f(\underline{\pi} - r)\left[\frac{\partial \underline{\pi}}{\partial \beta} - \frac{\partial r^{\beta}}{\partial \beta}\right] - f(\overline{\pi}+r)\left[\frac{\partial \overline{\pi}}{\partial \beta} + \frac{\partial r^{\beta}}{\partial \beta}\right].$$

To determine the sign of this expression, note that using (A-41) we obtain

$$\frac{\partial \overline{\pi}}{\partial \beta} + \frac{\partial r^{\beta}}{\partial \beta} = -\frac{A\int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} \left[f(\overline{\pi}+r^{\beta}) - f(x)\right] dx + (A\beta-1)\left[f(\overline{\pi}+r^{\beta}) - f(\overline{\pi})\right]\frac{\partial r^{\beta}}{\partial \beta}}{-(A\beta-1)\left[f(\overline{\pi}+r^{\beta}) - f(\overline{\pi})\right] + \left[r^{\beta}(A\beta-1)+\delta\right] f'(\overline{\pi}+r^{\beta})}. \quad \text{(A-42)}$$

Assumption 1 ensures that the integral term in the numerator as well as $f(\overline{\pi}+r^{\beta}) - f(\overline{\pi})$ are both negative. Since by assumption, $A\beta > 1$ (otherwise there is no interior solution to the policymaker's problem) and since by Lemma 4, $\frac{\partial r^{\beta}}{\partial \beta} > 0$, it follows that the numerator is negative. The denominator is also negative by the second order conditions for maximization. Hence, $\frac{\partial \overline{\pi}}{\partial \beta} + \frac{\partial r^{\beta}}{\partial \beta} < 0$. Similar calculations establish that $\frac{\partial \underline{\pi}}{\partial \beta} - \frac{\partial r^{\beta}}{\partial \beta} > 0$. Hence, $\frac{\partial P^{\beta}}{\partial \beta} > 0$, implying that as the policymaker's reputation improves, there is a greater likelihood of speculative attacks. **Q.E.D.**

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