The permanent-transitory confusion: Implications for tests of market efficiency and for expected inflation during turbulent and tranquil times

Alex Cukierman, Thomas Lustenberger and Allan Meltzer

6 August 2018

Personal note on the history of the paper by Alex Cukierman

The origin of this paper goes back to an old unpublished manuscript by Cukierman & Meltzer (1982). A couple of years before that time Karl Brunner and Allan Meltzer became aware of the importance of the permanent-transitory confusion. I first discovered the universality of this confusion for the formation of expectations and for economic behavior when, as a visiting scholar at Carnegie-Mellon during the end of the seventies and beginning of the eighties , I started to interact with Allan and Karl on this topic. This interaction culminated in a number of joint published papers. The Cukierman & Meltzer (1982) paper was a later spinoff of this research effort and was never completed mainly because the research attention of both Allan and myself had turned to other topics and I had returned to Tel-Aviv.

But I always felt that the ideas in our unpublished manuscript are sufficiently important to justify bringing it up to date and amplifying its message with empirical work. This is particularly important for younger generations of economists who, due to the early criticism by the rational expectation school that adaptive expectations are not rational, might not be aware of the fact that Muth (1960) provided a statistical foundation for the permanent-transitory confusion in which adaptive expectations are rational.

When the organizers of the conference on "Expectations: Theory and applications in historical perspectives" suggested I write a paper for the conference I felt the time had come to do that. Unfortunately on May 8 2017 my long time friend and collaborator Allan Meltzer passed away. My young collaborator, Thomas Lustenberger joined me in this effort. The current updated and

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Alex Cukierman
Tel-Aviv University, interdisciplinary Center and CEPR,
e-mail: alexcuk@tauex.tau.ac.il

Thomas Lustenberger
University of Basel and Swiss National Bank,
e-mail: thomas.lustenberger@unibas.ch

Allan Meltzer
Carnegie-Mellon University. Allan Meltzer passed away on May 8 2017.

expanded version of the paper integrates the analysis in the original manuscript with some newer empirical work. Beyond the personal loss I feel along with Allan's family the history of the paper is one illustration of the process through which the research torch is being passed across generations of economists. I view it as a personal token of appreciation to Allan Meltzer who had a major influence on my professional life.

Abstract Even when all past and present information is known individuals usually remain uncertain about the permanence of observed variables. After reviewing the history and role of adaptive expectations and its statistical foundations in modeling this permanent-transitory confusion the paper investigates the consequences of this confusion for tests of market efficiency in the treasury bill and foreign exchange markets. A central result is that the detection of serial correlation in efficiency tests based on finite samples does not necessarily imply that markets are inefficient.
The second part of the paper utilizes data on Israeli inflation expectations from the capital market to estimate the implicit speed of learning about changes in inflation and to examine the performance of adaptive expectations in tracking the evolution of those expectations during the 1985 shock stabilization as well as during the stable inflation targeting period.

1 Introduction

In making decisions economic agents and policymakers have to form expectations about the future. The importance of expectations is pervasive and of paramount importance for current decision making. Following are some well known illustrations: When deciding how to allocate an increase in income between consumption and savings individuals need to evaluate the permanence of this increase. A worker's decision about whether to accept or reject a poor job offer depends on his perception about the permanence of this condition. A firm's investment decision following strong demand for its product depends on its perception of the persistence of this state. When confronted with a strong economy monetary policymakers may consider an increase in the policy rate. But if they believe the strength is temporary they are likely to postpone the increase. Similar considerations apply to contractionary fiscal policies.

Generally, even when they possess full information about current and past realizations of relevant variables, individuals remain uncertain about their permanence. In many cases individuals detect the permanence of changes by observing the persistence of those changes over time. As a consequence when permanent changes occur they are recognized only gradually. Adaptive expectations capture this sluggishness by making the difference between the current and the previous period's forecasts a positive function of the forecast error committed in the previous period. Muth (1960) has shown that when a stochastic variable is composed of a random walk and a white noise process, none of which is ever observed separately, adaptive expectations are rational in the sense that they utilize all available information in an efficient manner. For brevity the paper refers to this residual uncertainty as the "permanent-transitory confusion" (PTC).

The first part of the paper reviews the history and past applications of Muth type adaptive expectations and considers their implications for standard tests of market efficiency. Using data on inflationary expectations from the Israeli capital market the second part examines the performance of Muth's model in tracking those expectations during the turbulent 1985 Israeli stabilization as well as during the stable 2003-2018 period.

Tests of efficiency in the treasury bill market as predictors of inflation proceed by regressing the current realization of inflation on a lagged capital market variable that embody the preceding period's expectation of inflation. Relying on Fisher's theory of interest this signaling variable is taken to be the lagged value of the nominal interest rate. In tests of efficiency of foreign exchange markets the signaling variable is taken to be the forward exchange rate leading to formulations in which the current rate of change in the exchange rate is regressed on the rate of change implied by the past forward rate. In either case the appearance of serial correlation in the residuals of those regressions is considered as evidence against market efficiency. The intuition supporting this view is that, if markets were efficient, rational individuals should have used it in their predictions leading to the disappearance of serial correlation. A central result of the first part of this paper is that, in the presence of the permanent-transitory confusion, the appearance of serial correlation in finite samples does not necessarily imply that markets are inefficient.

The paper is organized as follows. Section 2 reviews the history and past applications of adaptive expectations and presents Muth's (1960) statistical foundations for it. Section 3 reviews standard tests of market efficiency in the treasury bill and in the foreign exchange markets. Section 4 contains a main result of the paper. It proposes a generalization of the tests in section 3 and uses it to show that, following the realization of large permanent changes, the appearance of serial correlation in the residuals of the regressions used to implement those tests does not necessarily indicate that markets are inefficient.

An attractive feature of Muth (1960) foundation for adaptive expectations is that it relates the speed of learning about permanent changes to the relative size of the variability of the permanent component of a shock to the variability of the transitory component of the shock. Sections 5 and 6 use data on Israeli inflation expectations from the capital market along with this relation in order to estimate the implicit speed of learning about changes in inflation and to examine the performance of Muth's adaptive expectations model in tracking the evolution of capital market expectations. Section 5 focuses on the period before and after the 1985 stabilization that led, after a while, to a substantial decrease in expected inflation. The numerical exercise suggest that adaptive expectations provide a good approximation for the evolution of capital market expectations during this period. Section 6 applies a similar methodology to the stable inflation targeting period between 2003 and 2018. The numerical exercise supports the conclusion that, during this period, capital market participants considered all deviations from the inflation target as transitory. This is followed by concluding remarks.

2 Adaptive expectations through the ages and Muth model of the permanent-transitory confusion

2.1 Adaptive expectations

Adaptive expectations have been around for over a century. Although their roots go back to Irving Fisher (1911) they gained prominence and became operational in macroeconomics with the empirical work of Cagan (1956) on hyperinflations during the twentieth century and Friedman (1957) research on the permanent income hypothesis. Cagan used adaptive expectations to characterize the links between actual inflation in the past and inflationary expectations during the hyperinflation. Friedman applied them to model and estimate the links between perceived future permanent income and past realizations of actual income.

The basic idea of adaptive expectations is quite intuitive. It states that, when new information about a variable that is being forecasted becomes available over time, individuals adjust their expectations about the future realization of this variable in proportion to the forecast error committed in the previous period. For this reason the process is also frequently characterized as an ”error correction process”. Formally, adaptive expectations are given by

$$y_t^e - y_{t-1}^e = \theta(y_t - y_{t-1}^e) \tag{1}$$

where y_t is the actual realization of a variable y in period t and y_t^e is the forecast of that variable given the information available in period t. The adaptive expectations coefficient, θ, characterizes the speed with which the public incorporates recent developments into its forward looking expectations. In empirical applications θ is usually assumed to be bounded between 0 and 1. Moving y_{t-1}^e to the right hand side, lagging by one period in order to express y_{t-1}^e in terms of y_{t-1} and y_{t-2}^e, inserting the resulting expression into equation (1), and repeating this procedure ad-infinitum y_t^e can be rewritten in the integral form

$$y_t^e = \sum_{i=0}^{\infty} \theta(1-\theta)^i y_{t-i}. \tag{2}$$

With the onset of the rational expectations revolution Lucas (1972) and others criticized adaptive expectations on the ground that they were backward rather than forward looking. Rational expectations imply that

$$y_t^e \equiv E_t y_{t+1}$$

where $E_t y_{t+1}$ is the expected value of y_{t+1} given the information available up to and including period t. As shown in the next subsection, and as recognized later, the criticism above is not justified in the presence of the permanent-transitory confusion.

2.2 *The permanent-transitory confusion and Muth (1960) statistical foundations for it.*

The permanent-transitory confusion (PTC) refers to the widespread fact that knowledge of current and past changes in a stochastic variable normally leaves a margin of uncertainty about how much of those changes will persist into the future and how much are just temporary changes that will fade away as the future unfolds. The PTC is a pervasive fact of life that confronts investors, consumers, producers and policy-makers when they make current decisions. In a path breaking article Muth (1960) developed the following stylized statistical model for the PTC.[1] The model postulates that the stochastic variable, y_t, is the sum of two stochastic components none of which is ever observed separately. One is a random walk that persists into the future and the other is a transitory white noise that appears in period t and does not persist at all into the future. More formally

$$\begin{aligned} y_t &= y_t^p + y_t^q, \\ &\Delta y_t^p \sim N(0, \sigma_p^2), \\ &y_t^q \sim N(0, \sigma_q^2), \\ &\Delta y_t^p \text{ and } y_t^q \text{ are mutually independent.} \end{aligned} \tag{3}$$

Here Δy_t^p is the first difference of the random walk (permanent) component and y_t^q is the white noise (transitory) component. Muth (1960) has shown that the **forward looking** optimal predictor of $y_{t+j},\ j \geq 1$ given the information set, $I_t \equiv \{y_t, y_{t-1}, y_{t-2}, ...\}$, available in period t is identical to the adaptive expectation process in equations (1) and (2).[2] Furthermore the coefficient θ is an increasing function of the ratio, a, between the variance, σ_p^2, of the innovation to the random walk component and the transitory variance, σ_q^2 and is given by

$$\theta = \sqrt{a + \frac{a^2}{4}} - \frac{a}{2},\ a \equiv \frac{\sigma_p^2}{\sigma_q^2}. \tag{4}$$

Muth's optimal predictor has some notable and convenient features that are briefly summarized in what follows. First it implies that it is optimal to utilize **all past** observations on y_t in order to forecast the **future.** Second, equation (2) implies that it is a Koyck lag with geometric weights that decrease the more distant in the past is the observation on y. Third, the weights sum up to one. Fourth, the larger is the adaptive expectations coefficient, θ, the larger is the sum of the weights on the most

[1] Although this article is relatively less known (and quoted) than Muth (1961) Econometrica article that inspired the rational expectations revolution in macroeconomics its contribution is, nonetheless, not less important.

[2] Statistically minded readers may note that this optimal predictor is the expected value of y_{t+j}, $j \geq 1$ conditional on the information set, $I_t \equiv \{y_t, y_{t-1}, y_{t-2}, ...\}$. Due to the normality assumption this conditional expected value is linear in the elements of the conditioning set and the weights are those that minimize the variance of forecasts around this expected value.

recent past in comparison to the more distant past. Consequently, the larger is θ, the faster is the speed at which individuals detect a permanent change when such a change has occurred implying that θ characterizes the speed of learning. Finally, it is not surprising that θ and the ratio, a, between the permanent and the transitory variances are positively related. The higher is a, the higher is the signal to noise ratio implying that optimal learning should be faster.

The more general message of the preceding discussion is that, although predictors of the future are forward looking, they normally rely on past information since the past contains useful, albeit imperfect, information about the future. During the early days of the rational expectations revolution some economists criticized adaptive expectations on the ground that they are backward rather than forward looking. This criticism is probably based on perfect foresight models like that of Barro & Gordon (1983) that do not feature stochastic terms. In such models rational expectations reduce to the, known with certainty, values of relevant variables as predicted by such models. But once the more realistic existence of stochastic terms and the PTC are incorporated into models the role of **past information in predicting the future** becomes essential. Muth's predictor provided an early convenient way to capture the main features of the PTC and to relate it to natural intuition. But it is by no means, the only way to do that. A multi-variables generalization is provided by the Kalman Filter (Kalman (1960)).[3]

2.3 Past applications of Muth's predictor

Lucas & Rapping (1969) develop a model of employment/unemployment in which individuals decide how much of their employment efforts to allocate to the present versus the future. This decision is based on a comparison of their current wage with what they believe is their long run normal or permanent wage rate. Brunner et al. (1980) embed this mechanism along with Friedman's permanent income hypothesis into an extended IS-LM model. They utilize Muth's predictor to characterize the behavior of individual expectations about permanent income and permanent wages. Cukierman (1982) uses it to investigate the behavior of relative prices and of the allocative efficiency of the price system in the presence of the PTC about individual prices in a Lucas (1973) type multi-markets model.

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[3] A compact useful presentation of the Kalman Filter appears in chapter 21 of Ljungqvist and Sargent (2000).

3 Tests of market efficiency in the treasury bills and foreign exchange markets

To test for the efficiency of short term treasury bill rates as predictors of future inflation Fama (1975) relied on Fisher's (1930) theory of interest according to which those rates reflect the sum of the equilibrium real interest rate and the rate of change in the real value of money expected to realize over the life of the bill. The efficient markets or rational expectations hypothesis implies that in a linear regression of the rate of change in the real value of money on a previous market forecast of this change there should be no correlation in the residuals. Sample evidence of serial correlation in the residuals is taken to imply that individuals do not utilize all currently available information in an efficient manner since errors of forecast can be reduced by using the information contained in the persistent deviations of actual values from the forecast values implied by observable market values.

The simplicity of the test and the intuitive appeal of Fama's interpretation led to its application in other asset markets. Hamburger & Platt (1975) used current values of forward rates on treasury bills to forecast future spot rates. They found evidence of positive serial correlation in the residuals from some of their regressions and corrected for this "inefficiency" using the first order Cochrane-Orcutt procedure. Frenkel (1977, 1979) and many others subsequently used very similar procedures to test for the efficiency of forward rates as predictors of future spot exchange rates. Figlewski & Wachtel (1981) tested the rationality of individual price expectations by checking whether forecast errors are serially correlated and found those errors to be serially correlated. They concluded that survey respondents did not use all available information and that, consequently, the rational expectations (RE) hypothesis is violated.

3.1 Fama (1975) early efficiency test of current interest rates as predictors of future inflation

Fama (1975) tested the efficiency of one month treasury bills (TB) as predictors of the decrease in the real value of money over the remaining life of a bill as follows. The starting point of the test is the theory by Fisher (1930) according to which

$$\Delta_{t+1} = -r_t + R_t \tag{5}$$

where Δ_{t+1}, R_t and r_t are the decrease in the real value of money between month t and month $t+1$, the nominal and real rates at time t respectively and second order terms have been dropped. R_t is observed on the market at time t but Δ_{t+1} and r_t are stochastic variables at that time. The test consists in running the regression

$$\Delta_{t+1} = \alpha_0 + \alpha_1 R_t + \varepsilon_t \tag{6}$$

Under rational expectations cum risk neutrality and the additional assumption that the real rate is constant the hypothesis that short term nominal rates are efficient predictors of the upcoming monthly inflation reduces to a test of the joint hypothesis that $\alpha_1 = 1$, $\varepsilon_t = \Delta_{t+1} - E_t\Delta_{t+1}$ is a serially uncorrelated forecast error and α_0 is an estimate of minus the (assumed) constant real rate.[4] Note, in particular, that detection of serial correlation is taken as evidence against market efficiency. In Fama's words (1975, p. 273):
"*Nonzero autocorrelations imply that the market is inefficient; one can improve on the market's assessment of the expected value of* Δ_{t+1} *by making correct use of information in past values of* Δ_t".

3.2 *Efficiency tests of forward premia as predictors of future spot exchange rates*

The forward premium is the difference between the current forward and spot exchange rates. Similarly to the case of nominal rates as predictors of future inflation efficiency tests in the foreign exchange market are based on the notion that current forward market quotations embody expectations about future spot rates. Provided expectations are rational and market participants are risk neutral the forward premium should provide an unbiased estimate of the current market assessment of the change in the spot rate between the future maturity period of the forward rate and the current spot. More precisely, consider the regression of the change in the log of the spot exchange rate on the forward discount (expressed in log form)[5]

$$s_{t+1} - s_t = \alpha + \beta(f_t - s_t) + u_{t+1} \tag{7}$$

Here s_t is the log of the spot price of foreign currency at time t, f_t is the log of the one-period forward exchange rate at time t, and u_{t+1} is the regression disturbance. The general idea is that under risk neutrality and rational (or efficient) expectations the log of the forward rate provides an unbiased forecast of the log of the future spot exchange rate implying that u_{t+1} is a serially uncorrelated forecast error with zero mean. Translated into statistical hypothesis testing this implies the commonly tested null hypothesis that $\alpha = 0$, $\beta = 1$ and u_{t+1} has mean zero and is serially un-

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[4] Fama and others subsequently extended the test to fluctuating real rates. The central point of the next section applies to those extensions as well.

[5] For simplicity of exposition we focus in the text on the one period ahead forward premium as a predictor of the change in the exchange rate between the current and the next period. However all the discussion that follows in the text also applies to the k periods ahead forward premium. In this case equation (5) is simply replaced by

$$s_{t+k} - s_t = \alpha + \beta(f_{t+k} - s_t) + u_{t+k}.$$

correlated.[6] The intuition underlying the null hypothesis is that under risk neutrality and rational expectations the forward premium should equal the expected value as of period t of the spot rate in period $t+1$ implying that α should equal zero and that β should be equal to one.[7] Rational or efficient expectations also imply that u_{t+1} should have a zero mean and be serially uncorrelated since the contrary would imply that individuals do not efficiently utilize all the information available in period t violating the assumption of rational expectations.

3.3 A more general reformulation of market efficiency tests

The common feature in efficiency tests of short rates as predictors of inflation and of forward rates as predictors of the future rates is that, in both cases, currently observed market variables contain information about current expectations of future variables. This idea is captured more precisely by the following formalization

$$x_t = c_0 + c y_t^e \tag{8}$$

where x_t is a market variable observed at time t and c_0, c are constant coefficients that depend on the particular model under consideration. Solving for y_t^e in terms of x_t

$$y_t^e = -\frac{c_0}{c} + \frac{1}{c} x_t. \tag{9}$$

Consider the identity

$$y_{t+1} = y_t^e + (y_{t+1} - y_t^e). \tag{10}$$

Replacing the first y_t^e on the right hand side of this identity by equation (9)

$$y_{t+1} = -\frac{c_0}{c} + \frac{1}{c} x_t + (y_{t+1} - y_t^e) \tag{11}$$

we obtain a general formulation that subsumes the efficiency tests of the two preceding subsections as particular cases. It states that the realization of y_{t+1} is a linear function of period's t observed market variable, x_t, plus a forecast error, $y_{t+1} - y_t^e$. Efficiency of this more general model can be tested by running the regression

$$y_{t+1} = \beta_0 + \beta x_t + u_{t+1} \tag{12}$$

[6] This equation is the canonical regression used in the voluminous literature on the forward premium puzzle. See Chinn (2009), equation (2) and the adjoining discussion. Early formulations of the test were done in levels rather than in actual and expected rates of change (Frenkel (1977) and Frenkel (1979)).

[7] Subsequent literature such as Fama (1984) recognized the potential existence of risk aversion by introducing a risk premium into regression (7). A survey of this literature appears in Engel (1996). The central point of the next section applies also to formulations of equation (7) that incorporate a premium.

and by testing the restrictions on β_0, β and u_{t+1} implied by market efficiency for each of the models subsumed under the general formulation in equation (11).

When $y_{t+1} = \Delta_{t+1}$, $x_t = R_t$ and $c = 1$ equation (11) reduces to the regression used by Fama to test the efficiency of current rates in predicting future inflation (compared to equation (6)).[8]

When $y_{t+1} = s_{t+1}, x_t = f_t, c_0 = 0$ and $c = 1$ equation (11) reduces to the canonical regression used to test the efficiency of the forward premium in predicting future spot exchange rates (compare to equation (7)).

4 The impact of occasionally large permanent shocks on the serial correlation in forecast errors: The case of finite samples

Using Muth (1960) type optimal adaptive expectations this section shows that in finite samples that are occasionally subject to the realization of relatively large permanent shocks, estimated forecast errors will be serially correlated even when expectations are rational and markets are efficient. The wider implication of this result is that detection of such serial correlation does not necessarily indicate that markets are inefficient. To demonstrate this statement we focus on the general formulation of tests of market efficiency (equation (11)) in the presence of Muth's specification of the PTC (equation (3)). Equation (2) along with the optimality of those expectations implies

$$y_t^e = E_t y_{t+1} = \sum_{i=0}^{\infty} \theta(1-\theta)^i y_{t-i}. \tag{13}$$

Period's $t+1$ forecast error is given by

$$u_{t+1} = y_{t+1} - \sum_{i=0}^{\infty} \theta(1-\theta)^i y_{t-i}. \tag{14}$$

Following simple but tedious algebraic manipulations the forecast error can be rewritten[9]

$$\begin{aligned} u_{t+1} &= \overbrace{y_{t+1}^q - \sum_{i=0}^{\infty} \theta(1-\theta)^i y_{t-i}^q}^{\equiv Q_{t+1}} + \overbrace{\sum_{i=0}^{\infty} (1-\theta)^i \Delta y_{t+1-i}^p}^{\equiv P_{t+1}} \\ &= Q_{t+1} + P_{t+1} \end{aligned} \tag{15}$$

The first two terms on the right hand side of this expression summarize the impact of period's $t+1$ transitory component and of all past transitory components on period's $t+1$ forecast error. The last term summarizes the impact of all past innovations to

[8] As was the case before the estimate of $\beta_0 = c_0$, provides an estimate of minus the (assumed) constant real rate of interest.

[9] Details appear in subsection 1 of the Appendix.

the permanent component up to and including period $t+1$ on this forecast error. An immediate consequence of equation (15) is that the current forecast error depends on **all** the past history of shocks to both the permanent and the transitory components of y. Since individuals never observe (not even ex post) the permanent and transitory components of y separately this should not come as a surprise.

Since the transitory shocks and the innovations to the permanent shocks have zero expected value and are serially and mutually independent

$$\mathrm{Cov}(u_{t+1.}u_t) = Eu_{t+1.}u_t = EQ_{t+1}Q_t + EP_{t+1}P_t \tag{16}$$

and

$$\mathrm{Var}(u_{t+1}) = E\left\{Q_{t+1} + P_{t+1}\right\}^2 = E\left\{Q_{t+1}\right\}^2 + E\left\{P_{t+1}\right\}^2 \tag{17}$$

where

$$Q_{t+1} \equiv y^q_{t+1} - \sum_{i=0}^{\infty} \theta(1-\theta)^i y^q_{t-i} \text{ and } P_{t+1} \equiv \sum_{i=0}^{\infty} (1-\theta)^i \Delta y^p_{t+1-i}. \tag{18}$$

It is shown in subsection 2 of the Appendix that, in spite of the infinite series of overlapping terms between $u_{t+1.}$and u_t, the first order covariance between those forecast errors in the population is zero.[10] But, when a relatively large permanent innovation occurs in a finite sample the covariance between adjacent forecast errors may be positive for a sufficiently long time to produce evidence in favor of first order serial correlation in spite of the fact that the predictor in equation (13) is optimal.

The reason is that the public is unable to fully identify permanent changes even after the fact. They learn gradually, but optimally, according to equation (13), by observing that y maintains a value that is greater (or lower) than expected for some time. If the learning parameter, θ, is sufficiently low econometricians that implement market efficiency tests may find evidence of serially correlated forecast errors in finite samples that are dominated by the realization of a large permanent shock.

To show that expost forecast errors appear to be serially correlated under the circumstances just described we focus on the coefficient of correlation between adjacent forecast errors following the realization in period t of a relatively large permanent innovation, Δy^p_t. In order to focus on the impact of a large permanent shock in comparison to the normal variabilities of both shocks we assume that all the other realizations of the transitory and permanent innovations are equal to their repective standard deviations. The formula for this conditional (on a large Δy^p_t) coefficient of correlation is

$$\rho_j(\Delta y^p_t) \equiv \frac{E\left\{u_{t+j+1.}u_{t+j} \mid \Delta y^p_t\right\}}{\sqrt{E(u_{t+j+1})^2 E(u_{t+j})^2}} \; j \geq 0 \tag{19}$$

where the symbol E stands for the expected value over the distributions of both the permanent and transitory shocks. It is shown in subsection 3 of the Appendix that this coefficient is given by

[10] It is likely that this is the case also for higher order covariances between forecast errors.

$$\rho_j(\Delta y_t^p) = (1-\theta)^{2(j+1)}\left[\frac{(\Delta y_t^p)^2}{\sigma_q^2} - \frac{\sigma_p^2}{\sigma_q^2}\right] \tag{20}$$

Note that, when the squared ratio of period's t permanent shock to the transitory variance is identical to the signal to noise ratio, $\frac{\sigma_p^2}{\sigma_q^2}$, $\rho_j(\Delta y_t^p)$ is zero. This provides a "normal" benchmark value for $\rho_j(\Delta y_t^p)$. But, following a large realization of this squared ratio in comparison to the signal to noise ratio $\rho_j(\Delta y_t^p)$ is positive.[11] Due to gradual learning it is largest in the period immediately following the realization of the large permanent shock. It then gradually declines to zero as the impact of the shock on current expectations fades into the past. When the learning parameter, θ, is relatively low (or equivalently $\frac{\sigma_p^2}{\sigma_q^2}$ is low) this positive sample correlation may persist for quite a while before it finally converges to its normal zero value. On the other hand the likelihood that a relatively large value of the permanent shock occurs is lower when $\frac{\sigma_p^2}{\sigma_q^2}$ is low.

The upshot is that, although the probability of a large realization of Δy_t^p is low when $\frac{\sigma_p^2}{\sigma_q^2}$ is low, if such a low probability event does occur, it induces in finite samples persistent measured serial correlation in forecast errors. Figure 1 illustrates the behavior of $\rho_j(\Delta y_t^p)$ for $\frac{(\Delta y_t^p)^2}{\sigma_q^2} = 3$ and $\theta = 0.01$. The figure shows that following the realization of this large permanent shock the covariance between forecast errors is larger than the variance of those errors ($\rho_j(\Delta y_t^p) > 1$) for over 50 periods after the realization of this shock in spite of the rationality of expectations. On the other hand once the speed of learning rises above 0.2 most of this persistence vanishes given the same value of $\frac{(\Delta y_t^p)^2}{\sigma_q^2}$.

The more general lesson from this exercise is that in tests of efficiency of the treasury bill market, the failure to reject serial correlation can be miss-leading if applied to samples taken shortly after violent changes in the purchasing power of money. Similarly, the serial correlation test may yield wrong conclusions about the efficiency of the foreign exchange market if applied during or shortly after large permanent changes in the exchange rate. Interestingly, Frankel & Poonawala (2006), Table II, reject the null hypothesis of no serial correlation in forecast errors at the 5% significance level for India, Indonesia and Turkey. Our analysis implies that this finding does not necessarily imply that foreign exchange markets in those countries are inefficient.

[11] Note that, since it depends on a particular realization of the innovation to the permanent component, $\rho_j(\Delta y_t^p)$ is not necessarily smaller than one.

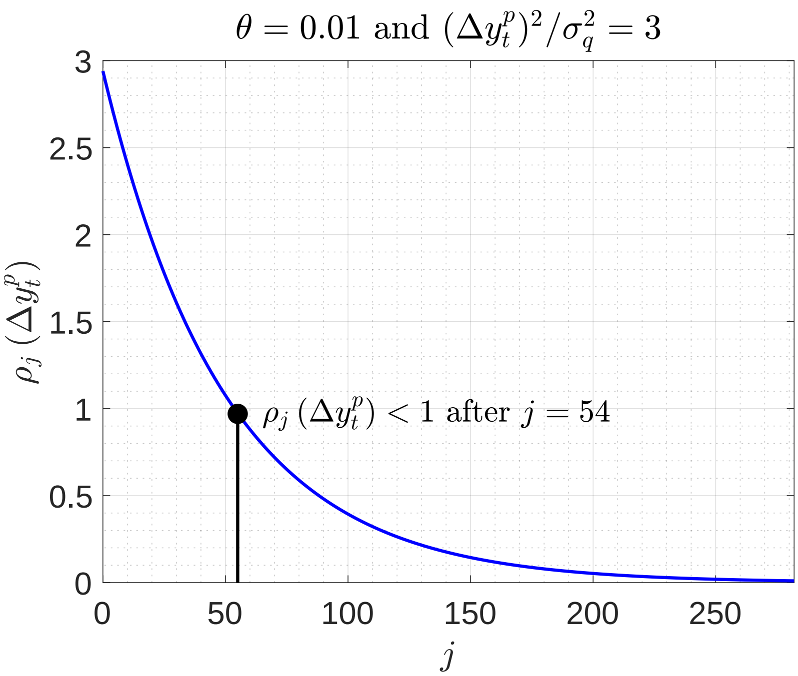

Fig. 1 Conditional coefficient of correlation

5 Turbulent times: The behavior of expected inflation during disinflation

Since about the mid nineties the Bank of Israel has been deriving estimates of expected inflation from the difference between the yields to maturity on indexed and non indexed government bonds. Due to the absence of long term nominal bonds at the start of the period those estimates, also known as breakeven inflationary expectations, were initially limited to forecast horizons of one year. But, as inflation subsided at the beginning of the twenty first century the Israeli treasury issued nominal bonds with longer maturities making it possible to derive longer term inflationary expectations from the bond market up to a horizon of ten years.

5.1 An empirical application to the Israeli 1985 cold turkey stabilization

A“cold turkey” or“shock” stabilization refers to a situation in which high inflation is stabilized very aggressively within a short period of time. Following seven years with yearly rates of inflation of 100 percent or more and several failed attempts to stabilize inflation Israel finally managed to stabilize it in July 1985 bringing the rate of inflation down from about 400 percent to almost zero within a couple of months. This dramatic drop was achieved through the simultaneous deployment of conventional measures like restrictive fiscal and monetary policies as well as less conventional measures such as temporary controls on prices, wages and the exchange rate.[12]

It can be concluded with the benefit of hindsight that the 1985 cold turkey stabilization produced a large permanent drop in the rate of inflation. However, at the time of the stabilization, there was substantial uncertainty about the extent to which this dramatic drop will persist. This uncertainty was induced by wide gyrations in inflation and several failed attempts to stabilize prior to the 1985 successful stabilization. It is therefore instructive to examine the behavior of inflationary expectations before and after the 1985 stabilization.

Although capital market inflationary expectations were not calculated on a systematic basis prior to the mid-nineties, they were occasionally estimated also prior to that time. In particular Table 2.2 in Cukierman (1988) provides average monthly breakeven expected inflation over a three month horizon along with average monthly inflation over the same horizon between January 1984 and October 1986. Figure 2 plots actual and previously expected average inflation at monthly rates for this period.

Perusal of the figure suggests that breakeven inflationary expectations lagged behind changes in the actual rate of inflation. It is likely, therefore, that this gradual adjustment of expectations indicates that expectations are adaptive and that Muth’s model of the PTC may provide a reasonable approximation to the behavior of actual inflation and of breakeven expectations during the time period displayed in the figure. To examine this possibility the next subsection utilizes the data on actual and expected inflation underlying Figure 2 to estimate the learning parameter, θ, and the variances, σ_p^2 and σ_q^2 of the permanent and transitory shocks to the components of inflation over this period.

5.2 Estimation of the learning parameter during the 1985 stabilization

Since observations on breakeven expectations are available the parameter θ that fits the data best can be estimated from equation (1) where y_t and y_t^e stand now for actual

[12] A detailed description of the 1985 stabilization appears in Bruno & Piterman (1988).

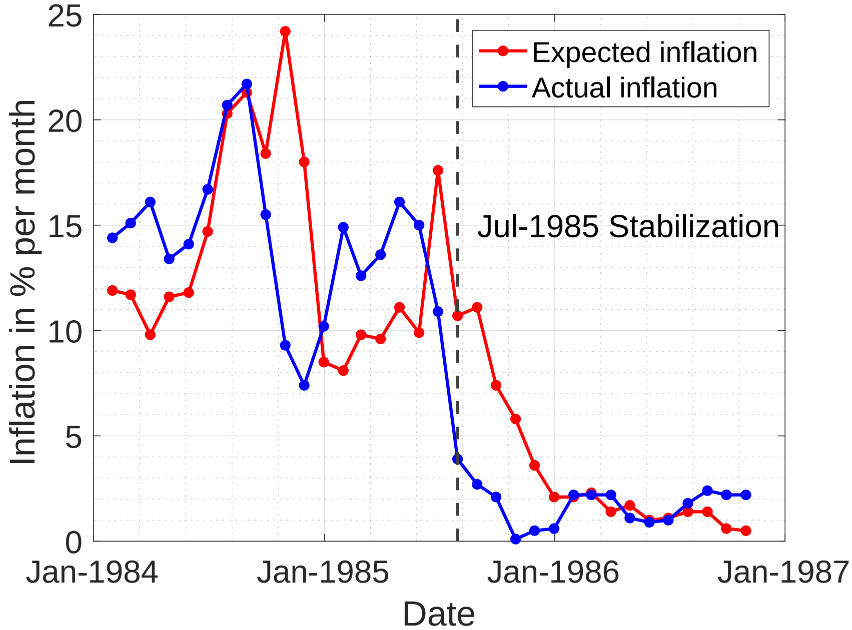

Fig. 2 Expected three months ahead and actual inflation from January 1984 to October 1986

and expected inflation. Estimation of σ_p^2 and of σ_q^2 requires the prior estimation of the variance of the first difference of actual inflation. Taking the first difference of y_t from equation (3)

$$\Delta y_t = \Delta y_t^p + y_t^q - y_{t-1}^q. \tag{21}$$

Equations (3) and (21) imply that the variance, $\sigma_{\Delta y}^2$, of Δy_t is

$$\sigma_{\Delta y}^2 = \sigma_p^2 + 2\sigma_q^2. \tag{22}$$

$\sigma_{\Delta y}^2$ is estimated by taking first differences of y_t and by calculating the variance of those differences over the sample period. It is shown in subsection 4 of the Appendix that equation (4) is equivalent to

$$a \equiv \frac{\sigma_p^2}{\sigma_q^2} = \frac{\theta^2}{1-\theta}. \tag{23}$$

Finally, given the estimates of θ and of $\sigma_{\Delta y}^2$ equations (22) and (23) are used to obtain estimates of σ_p^2 and of σ_q^2. The estimated values are $\theta = 0.32$, $\sigma_p^2 = 0.49$,

$\sigma_q^2 = 3,34$ implying that the signal to noise ratio, $a = \frac{\sigma_p^2}{\sigma_q^2}$, is 0.14. Given the estimate of the learning parameter, θ, simulated values of the breakeven expectations are calculated by using expected inflation and actual figures in equation (1).

Figure 3 shows simulated values of the three months ahead capital market expectations along with the actual values of those expectations. It is apparent from the figure that Muth's stochastic structure with $\theta = 0.32$ performs quite well in tracking actual values of those expectations particularly following the July 1985 stabilization. This conclusion is also backed by the finding that the ratio between the sum of squared deviations of simulated from actual values of expectations and the variance of actual expectations is only **0.15**. This evidence supports the conclusion that the stochastic structure postulated in Muth (1960) fits the data around the 1985 stabilization of inflation reasonably well. More precisely it implies that inflation during the 1984-1986 period can be characterized as the sum of a random walk and of a white noise (equation (3)).

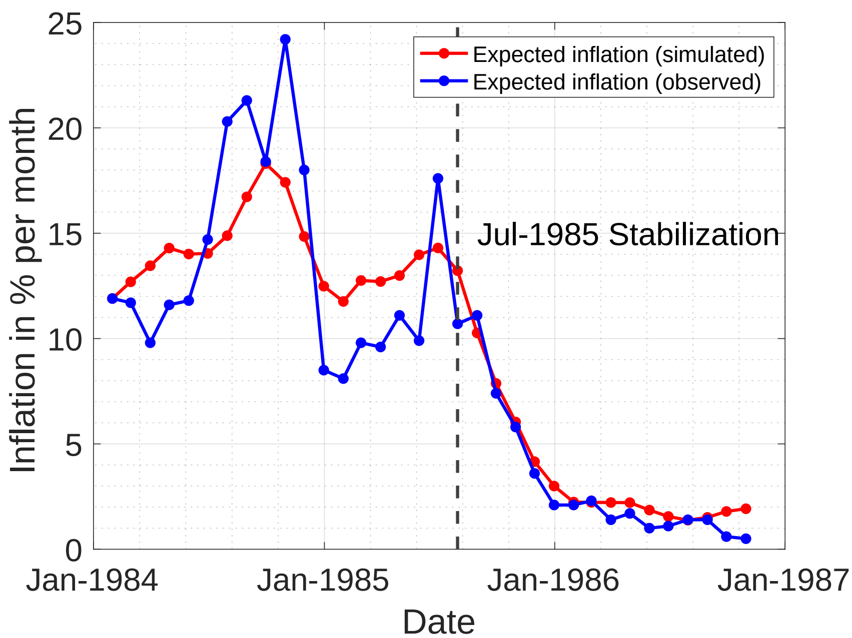

Fig. 3 Actual and simulated three months ahead inflation expectations from January 1984 to October 1986

6 The performance of adaptive expectations during tranquil times: Israel 2003-2018

Although the July 1985 stabilization permanently reduced inflation below 20 percent per year inflation converged to the vicinity of the 2 percent international standard only at the beginning of the twenty first century. An, initially informal, inflation target regime was inaugurated in the mid nineties. The target was first used as an instrument for reduction of expectations without excessive decreases in economic activity and was gradually decreased from year to year when the previous year's target was attained. It finally converged to a long run fixed inflation target of 2 percent central target with an allowable band between 1 and 3 percent at the beginning of 2003. From that point and on actual inflation remained most of the time within this target range.[13]

The main objective of this section is to examine empirically the ability of Muth's adaptive expectations model to provide a characterization of capital market expectations and to estimate the speed of learning, θ, during the 2003-2018 tranquil period. During this period 10 as well as one year ahead capital market inflationary expectations are available. Figure 4 shows actual and long term expected capital market expectations along with the fixed inflation target for this period.

6.1 Estimation of the learning parameter during the tranquil 2003-2018 period

The discussion in the section focusses on 10 years ahead expectations but results for a one year horizon are briefly reported as well. The methodology for estimation of the parameters θ, σ_p^2, σ_q^2 and a is similar to the estimation of those parameters in the turbulent period discussed in the previous section except for the fact that during the eighties there was no inflation target. To take into consideration the existence of a pre-announced fixed long term target, δ, during the tranquil period the actual rate of inflation (denoted now π_t) is respecified as

$$\begin{aligned} \pi_t &= \delta + y_t = \delta + y_t^p + y_t^q \\ \pi_t^e &= \delta + y_t^e \end{aligned} \tag{24}$$

where the stochastic properties of y_t and of its constituent components are given in equation (3). That is, actual inflation is equal to a full certainty known in advance permanent target plus a stochastic deviation, y_t, that possesses the stochastic properties postulated by Muth to describe the PTC. Consequently the optimal forecast of π_t^e is given by the second line in equation (24). Rearranging equations (24)

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[13] A detailed description of the convergence process appear and other details appear in Cukierman & Melnick (2015).

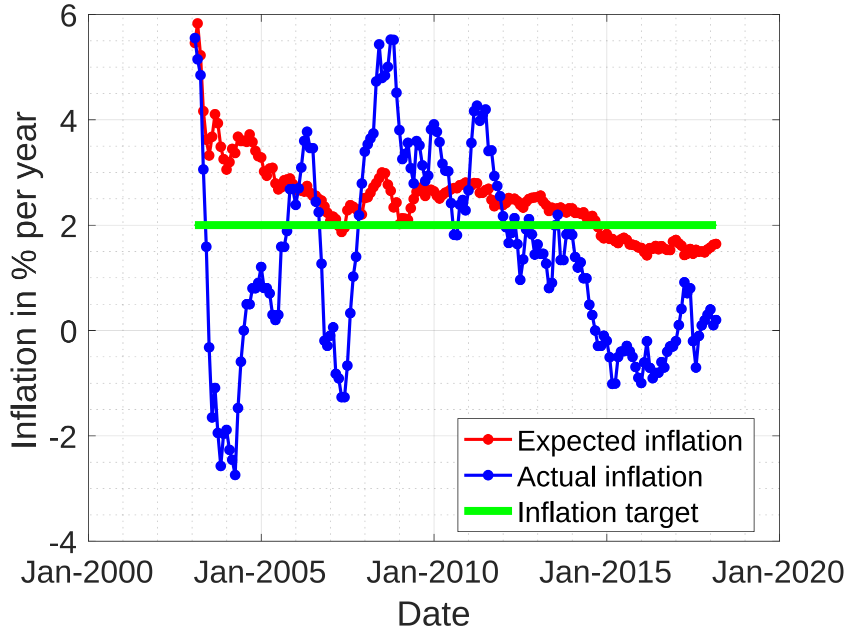

Fig. 4 Expected ten years ahead and actual inflation from from January 2003 to February 2018

$$\begin{aligned} y_t &= \pi_t - \delta \\ y_t^e &= \pi_t^e - \delta. \end{aligned} \tag{25}$$

Since y_t has the same stochastic properties as in the previous section the parameters θ, σ_p^2, σ_q^2 and a can be estimated by applying the procedure from that section to $\pi_t^e - \delta$. It is shown in subsection 5 of the Appendix that the estimates of θ and of $\sigma_{\Delta y}^2$ obtained by using y_t and y_t^e are identical to the estimates using the original actual and expected inflation figures π_t and π_t^e. Hence the procedure used in the previous section for estimation purposes can be applied directly to the expected inflation figures before the transformations in equations (25).

The estimated values are $\theta = 0.01$, $\sigma_p^2 = 0.00002$, $\sigma_q^2 = 0.13$ implying that the signal to noise ratio, $a = \frac{\sigma_p^2}{\sigma_q^2}$, is 0.0001. In sharp contrast to the turbulent 1985 high inflation period the variance of the stochastic permanent component is almost zero implying that the speed of learning about this component is extremely slow. In other words, during the tranquil period capital market participants practically considered all deviations from the long term 2 percent inflation target as transitory supporting the view that long term inflationary expectations were well anchored to the 2 percent target. Figure 5 shows actual and simulated values of long term expectations.

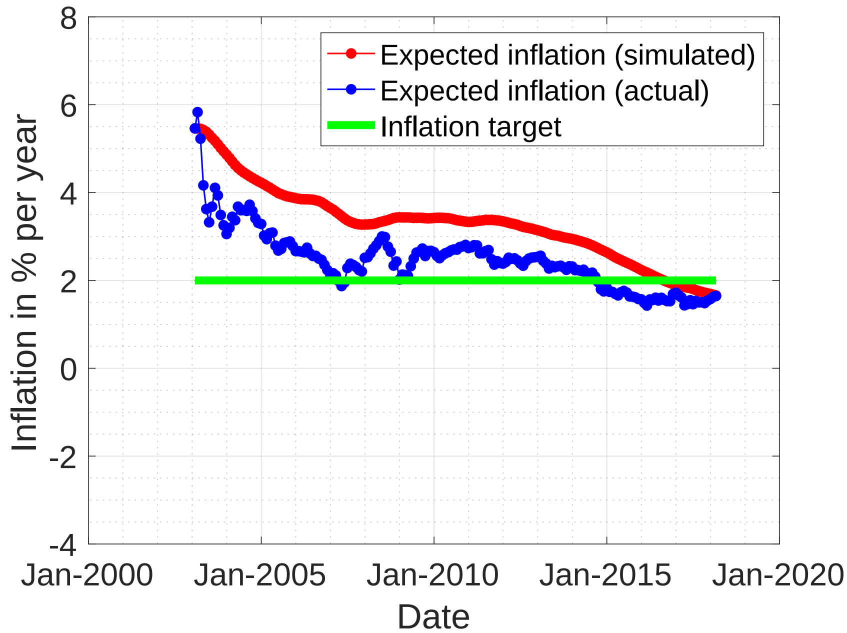

Fig. 5 Actual and simulated ten years ahead inflation expectations from January 2003 to February 2018

The ultimate emergence of a well maintained inflation targeting regime neutralized the impact of the stochastic PTC on long term expectations replacing it by a non-stochastic permanent inflation target of 2 percent implying that Muth (1960) process does a poor job of characterizing the behavior of ten years expectations data. This conclusion is backed by the finding that the ratio between the sum of squared deviations of simulated from actual values of expectations and the variance of actual expectations is a huge **1.62**.

Estimation results for one year ahead inflation expectation are broadly similar except that the speed of learning and the signal to noise ratio are somewhat higher. They are $\theta = 0.02$, $\sigma_p^2 = 0.00005$, $\sigma_q^2 = 0.13$ implying that the signal to noise ratio, $a = \frac{\sigma_p^2}{\sigma_q^2}$, is 0.0004. But, due to an almost doubling of the speed of learning the fit of simulated expectations is better than in the case of ten years ahead expectations. This is reflected in the finding that the ratio between the sum of squared deviations of simulated from actual values of expectations and the variance of actual expectations for the one years ahead expectations drops to **0.84**.

The upshot from this experiment is that, in contrast to the turbulent period, Muth (1960) stochastic assumptions and optimal predictor does not capture the behavior of both long and short term inflationary expectations well during the tranquil period. Instead it supports the view that capital markets participants considered all deviations for the inflation target as transitory. The broader consequences of this finding are discussed in the concluding section.

7 Concluding remarks

This paper reviewed the history of adaptive expectations as a vehicle for modeling the permanent-transitory confusion. A central result of the first part of the paper is that, in the presence of this confusion, the appearance of serial correlation in tests of market efficiency based on finite samples does not necessarily imply that markets are inefficient. This implies that the detection of serial correlation in tests of efficiency in the treasury bill and in the foreign exchange markets does not necessarily imply that the expectations embodied in interest rates and in forward exchange rates are not rational in the sense that they disregard relevant information.

Although the early rational expectations literature criticized adaptive expectations on the ground that they are backward rather than forward looking the work of Muth (1960) demonstrated that in the presence of the permanent-transitory confusion the optimal forecast of the future relies on information from the past. Muth considered only the case of a single stochastic variable in which the permanent component is a random walk and the transitory component is a white noise process. But the work of Kalman (1960) on the Kalman filter and subsequent literature suggest that, generally, optimal forecasts of the future rely on available past and current information and that this statement is true for a large class of more general processes that include both stationary and non stationary stochastic processes.[14] The crucial feature underlying this regularity is that the stochastic variables considered are composed of shocks with different degrees of persistence none of which is observed separately.[15]

Using Israeli data on inflationary expectations from the capital market the second part of the paper examines the performance of adaptive expectations in tracking those expectations during the 1985 Israeli stabilization as well as during the tranquil stable inflation targeting period. Adaptive expectations perform quite well prior to and shortly after the cold turkey 1985 stabilization but not during the tranquil inflation targeting period (2003-2018).

[14] One example is chapter 21 of Ljungqvist & Sargent (2000).

[15] Furthermore, as demonstrated by Friedman (1979) serial correlation may also arise when a slope coefficient of an economic model changes permanently. The reason is that an econometrician using least square becomes aware of the change only gradually as post change observations cumulate over time

As a matter of fact in the latter period the empirical results are consistent with the view that individuals in the capital market believed that the long run inflation rate is given by the two percent pre-announced stable inflation target and interpreted any deviation of inflation from this target as temporary. The wider economic implication is that, during the stable inflation targeting period capital market expectations were well anchored.[16] At the technical level this suggests that an expectation process in which the only permanent component is the pre-announced fixed inflation target and the temporary component is stationary is likely to produce a better fit for modeling the behavior of capital market expectations.

Acknowledgements We benefited from useful discussions on tests of market efficiency in the foreign exchange market with Menzie Chinn. The views expressed in this paper are those of the author(s) and do not necessarily reflect those of the SNB.

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[16] This conclusion is consistent with results obtained in Cukierman & Melnick (2015).

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8 Appendix

8.1 Derivation of equation (15)

Inserting equation (3) into equation (14) for all t

$$u_{t+1} = y^p_{t+1} + y^q_{t+1} - \sum_{i=0}^{\infty} \theta (1-\theta)^i (y^p_{t-i} + y^q_{t-i}).$$

Grouping all the transitory terms into one expression and all the first differences of the random walk component into another expression and rearranging

$$u_{t+1} = Q_{t+1} + P_{t+1}$$

where Q_{t+1} and P_{t+1} are given by equation (18) in the text. QED

8.2 Proof that $Eu_{t+1}u_t = 0$

It is convenient to first prove the following Lemma

Lemma 1. $(1-\theta)\sigma_p^2 - \theta^2\sigma_q^2 = 0$

Proof. Rearranging equation (4) in the text

$$\theta + \frac{a}{2} = \sqrt{a + \frac{a^2}{4}}.$$

Raising both sides of this equation to second power, cancelling terms and noting that $a \equiv \frac{\sigma_p^2}{\sigma_q^2}$

$$\theta^2 = \frac{\sigma_p^2}{\sigma_q^2}(1-\theta).$$

The proof is completed by moving σ_q^2 to the left hand side of this equation.

Since all the terms in P_{t+1} are statistically independent from the terms in Q_{t+1}

$$Eu_{t+1}u_t = EQ_{t+1}Q_t + EP_{t+1}P_t. \tag{26}$$

Using the definitions of Q_{t+1} and of P_{t+1} from equation (18) in the text it can be shown after some tedious algebra that

$$EQ_{t+1}Q_t = -\frac{\theta\sigma_q^2}{2-\theta}, \tag{27}$$

$$EP_{t+1}P_t = \frac{\sigma_p^2(1-\theta)}{\theta(2-\theta)}. \tag{28}$$

Substituting those expressions into equation (26)

$$Eu_{t+1}u_t = \frac{(1-\theta)\sigma_p^2 - \theta^2\sigma_q^2}{\theta(2-\theta)}.$$

By Lemma 1 the numerator of this expression is zero. Since the denominator is positive $Eu_{t+1}u_t = 0$. QED

8.3 Derivation of $\rho_j(\Delta y_t^p)$ (equation (20))

From equation (15) in the text

$$E\left[u_{t+j+1}u_{t+j} \mid \Delta y_t^p\right] = EQ_{t+1}Q_t + E\left[P_{t+j+1}P_{t+j} \mid \Delta y_t^p\right] \tag{29}$$

where

$$\begin{aligned} E\left[P_{t+j+1}P_{t+j} \mid \Delta y_t^p\right] = E&\left\{\Delta y_{t+j+1}^p + (1-\theta)\Delta y_{t+j}^p + ..\right\} \\ &\left\{\Delta y_{t+j}^p + (1-\theta)\Delta y_{t+j-1}^p + ..\right\} \\ &+(1-\theta)^{2j+1}\left\{(\Delta y_t^p)^2 - \sigma_p^2\right\}. \end{aligned} \tag{30}$$

Taking the expected value of the product in equation (30), summing up the resulting infinite series and rearranging this equation reduces to

$$E\left[P_{t+j+1}P_{t+j} \mid \Delta y_t^p\right] = \frac{(1-\theta)\sigma_p^2}{\theta(2-\theta)} + (1-\theta)^{2j+1}\left\{(\Delta y_t^p)^2 - \sigma_p^2\right\}. \tag{31}$$

Substituting equations (28) and (30) into equation (29), rearranging and using Lemma 1

$$E\left[u_{t+j+1}u_{t+j} \mid \Delta y_t^p\right] = (1-\theta)^{2j+1}\left\{(\Delta y_t^p)^2 - \sigma_p^2\right\}. \tag{32}$$

From equation (17) in the text

$$Eu_t^2 = EQ_t^2 + EP_t^2 \text{ for all } t. \tag{33}$$

Using the expressions for Q_t and P_t from equation (18) in equation (33), taking expectations of the resulting expressions, rearranging and using Lemma 1 yields

$$Eu_t^2 = \frac{\sigma_q^2}{1-\theta} \text{ for all } t.$$

Hence

$$\sqrt{E(u_{t+j+1})^2 E(u_{t+j})^2} = \frac{\sigma_q^2}{1-\theta}. \tag{34}$$

Equations (32) and equation (34) imply that

$$\rho_j(\Delta y_t^p) \equiv \frac{E\left\{u_{t+j+1}.u_{t+j} \mid \Delta y_t^p\right\}}{\sqrt{E(u_{t+j+1})^2 E(u_{t+j})^2}} = (1-\theta)^{2(j+1)}\left\{\frac{(\Delta y_t^p)^2}{\sigma_q^2} - \frac{\sigma_p^2}{\sigma_q^2}\right\}$$

QED

8.4 Derivation of equation (23)

The proof is an immediate consequence of Lemma 1. QED

8.5 Proof that using observations on y_t and y_t^e or on π_t and π_t^e yield identical estimates of θ and of $\sigma^2_{\Delta y_t^p}$

When the pair $\{\pi_t, \pi_t^e\}$ is used the estimate of θ is obtained by running the regression

$$\pi_t^e - \pi_{t-1}^e = \theta(\pi_t - \pi_{t-1}^e).$$

When the pair $\{y_t, y_t^e\}$ is used the estimate of θ is obtained by running the regression

$$y_t^e - y_{t-1}^e = \theta(y_t - y_{t-1}^e).$$

The definitions of $\{y_t, y_t^e\}$ in equation (25) in the text imply that the first and the second equations are identical so the estimate of θ obtained from either equation is the same.

When π_t is used to estimate $\sigma^2_{\Delta y_t}$ the estimate is the sample variance of $\pi_t - \pi_{t-1}$ and when y_t is used it is the sample variance of $y_t - y_{t-1}$. Since the definitions in equation (25) imply

$$y_t - y_{t-1} = \pi_t - \pi_{t-1}$$

the two estimates are identical. QED